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Production, Mortality, and Power Plant Entrainment of Larval Yellow Perch in Western Lake Erie

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PRODUCTION, MORTALITY, AND POWER PLANT ENTRAINMENT OF LARVAL YELLOW PERCH IN WESTERN LAKE ERIE

by

Richard L. Patterson
Large Lakes Research Station
Environmental Research Laboratory-Duluth
Grosse Ile, Michigan 48138

and

School of Natural Resources
University of Michigan
Ann Arbor, Michigan 48109

best copy available

ENVIRONMENTAL RESEARCH LABORATORY
OFFICE OF RESEARCH AND DEVELOPMENT
U.S. ENVIRONMENTAL PROTECTION AGENCY
DULUTH, MINNESOTA 55804

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FOREWORD

The Great Lakes with their large reservoir of water are subjected to many uses. Not only do they support abundant aquatic life, provide drinking water, recreation, transportation and waste assimilation, but their water is also used in industrial processes. Quite often it is very difficult to access the impact of one use upon another. This report has attempted to synthesize the data from many investigations into an analysis of the impact of the use of cooling water on the production of fish. It is through analyses such as those that have been performed in this report that we will be guided in making decisions on the management of the Great Lakes.

J. David Yount, Ph.D.
Deputy Director
Environmental Research Laboratory-Duluth

ABSTRACT

This study assessed impacts of the Monroe Power Plant upon the yellow perch population of Western Lake Erie caused by entrainment and impingement of larvae and older fishes in the plant's open cycle cooling system in 1975-76. Data analyzed in the study were collected by the Michigan Department of Natural Resources, the Center for Lake Erie Area Research of the Ohio State University, and the Institute of Water Research of Michigan State University. Estimates of total numbers of perch larvae entrained, total perch production, the natural mortality rate of perch, and the percentage of perch production that was entrained by the Monroe Power Plant were obtained for 1975-76. Impingement estimates were obtained from data supplied by the power plant. The above estimates consider only effects that occur in the same year in which entrainment and impingement occurs. Impacts may occur in subsequent years which include a depression of fish stocks and reduced yields to the fishery. Losses to the standing stocks and fisheries were estimated using a method which falls into a category known as the "equivalent-adult" type which provided estimates of the long-run annual depression of yellow perch standing stocks and the yellow perch fisheries. A numerical model was developed which incorporated several population parameters including entrainment and impingement losses, and natural mortality rates for larvae, young-of-year, and juveniles, and fishing mortality rates.

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SECTION 1

INTRODUCTION AND SUMMARY

The yellow perch population of Lake Erie has fluctuated widely over the past forty years as evidenced by commercial catch statistics and field surveys taken by the Ohio and Michigan Departments of Natural Resources and the U.S. Fish and Wildlife Service (12,19). Any occurrence of an increased juvenile recruitment rate (due to operation of compensatory factors associated with reproduction) has been insufficient to offset limiting factors such as higher fishing pressure (including pressure on yearlings), increased interspecific competition, and deterioration of the microhabitat of larvae and juvenile fishes. Field surveys show the occurrence of strong year classes only at very irregular intervals (17,19), between which these strong classes may be separated by as much as seven years. Strong year classes have repeatedly occurred, however, as the result of several interacting population and environmental factors, rendering the assignment of causes to fluctuations in year class sizes tentative at best. This is not to say that no relationships exist between reproduction, growth, standing crop, fishing, and natural mortality. It is all too well known that heavy natural predation combined with heavy fishing pressure will deplete Great Lakes fish stocks to the point of an irreversible decline. Power plants that employ open cycle, once-through cooling water systems are also known to cause losses of large numbers of larvae and young-of-year fishes by entrainment and impingement, although their impact upon the yellow perch fishery of Lake Erie has not been previously investigated. Eighteen municipal and industrial water intakes have been identified in Michigan-Ohio waters of the western basin of Lake Erie alone. Among these, the 3100 megawatt Detroit Edison power plant located at Monroe, Michigan has the largest water pumping capacity. Operating at 50 percent capacity for one 24 hour period, the Edison plant can pump approximately 4.32×10^6 cubic meters of water through the cooling cycle.

In order to assess the impacts that the Monroe power plant might be exerting upon the yellow perch population and fisheries of western Lake Erie, a three year field sampling program was undertaken to provide baseline data on larval perch abundance and entrainment levels.[1] The purposes of the

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[1]From late April through July in 1975 and 1976, biologists from the Michigan Department of Natural Resources (MDNR), the Institute of Water Research of Michigan State University (MSU), and the Center for Lake Erie Area Research (CLEAR) of the Ohio State University sampled larval fish densities throughout U.S. waters of the western basin. In 1977, the field observational program was conducted by CLEAR. Results reported in the present paper are based upon analyses of 1975-76 data only.

analyses of the data are as follows: 1. estimate production of larval yellow perch in Michigan-Ohio waters of the western basin; 2. estimate natural mortality of larval yellow perch prior to their recruitment into the young-of-year stage of development; 3. estimate the number of larval yellow perch entrained and killed in the cooling water cycle of the Monroe power plant; 4. estimate the percentage of total larval perch production in Michigan waters of the western basin that is lost in the cooling water cycle of the Monroe power plant; 5. estimate the percent loss in young-of-year recruitment attributable to entrainment mortality at the Monroe power plant; 6. estimate the loss to the yellow perch fisheries in western Lake Erie attributable to impingement and entrainment mortality occurring at the Monroe power plant.

Estimates of production and natural mortality of yellow perch larvae are obtained by formulating and solving a materials balance model of larval concentration (or abundance) which incorporates two parameters: h-total larval production in a season per 100 cubic meters of water in the reference volume; p-mean daily natural mortality rate. The model describes the time variation of mean larval concentration throughout the reference volumes (Michigan waters: 4.976×10^8 M^3, and Ohio waters: 9.393×10^9 M^3); the model parameters are estimated by the method of least squares.

Production of larval yellow perch in U.S. waters of the western basin in 1975 is estimated to have been $2.3x10^9$-$3.5x10^9$, of which $7.0x10^7$-$2.3x10^8$ are estimated to have survived for 25 days following hatching. Production in 1976 in U.S. waters of the western basin is estimated to have been $1.8x10^9$-$2.6x10^9$ of which $5.3x10^7$-$1.8x10^8$ are estimated to have been recruited into the young-of-year stage. Yellow perch larval production in Michigan waters in 1976 declined to approximately 27 to 29 percent of the 1975 level whereas production in Ohio waters declined to an estimated 83 to 85 percent of the 1975 level. When an estimated 50 percent survival of young-of-year fishes is combined with the an estimated 2 to 10 percent survival of larvae an estimated 1.0 to 5.0 percent of larvae survive to be recruited into the yearling stage of development.

The number of larvae estimated to have been lost due to entrainment at the Monroe power plant in 1975 is approximately $7.4x10^6$. The estimated number entrained, however, is nearly double that figure. The estimated yellow perch larval entrainment at the same plant in 1976 calculated by Detroit Edison personnel using their own pump samples is approximately 650,000. It is estimated that yellow perch larval losses attributable to the power plant in 1976 were between 195,000 and 2,827,000.

The percentage loss of recruitment of yellow perch into the young-of-year stage due to entrainment mortality at the Monroe plant is estimated to be 0.8 to 4.7 percent for 1975 and 0.9 to 1.5 percent in 1976, considering Michigan waters only.

It is estimated that the potential long run annual loss to commercial and sport fisheries is approximately 110,000 to 406,000 pounds. The above value is the best interval estimate obtainable and is the result of averaging the values given in Tables 25 through 32 for different combinations of population parameters and fishing mortality. The most basic assumption underlying the

analysis is that combined pressures on the yellow perch population will not be so severe as to exhaust the reproductive stock. The effects of compensatory mechanisms possibly operative in the yellow perch population are unknown, although the compensatory reserve is believed to be slight. The differential impact of entrainment and impingement losses may be greatest when the fishery is in a depressed condition, which is the present situation. The basic reason for this increased impact is that when the compensatory reserve is zero, low numbers of reproductive stock cannot replace incremental losses to that stock at all. Additional increments of loss in such a situation can drive the population into an irreversible decline. If the yellow perch fishery were tightly regulated, and if it rested upon a large reproductive base, reproductive compensation could conceivably account for most, if not all, of the losses caused through entrainment and impingement mortality incurred by cooling waters of the Edison power plant at Monroe.

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[1]Calculations in Appendix 8 indicate that annual losses could prove to be considerably higher.

SECTION 2

OBJECTIVES

The following analysis of field data collected in 1975-1976 is part of a program sponsored by the U.S. Environmental Protection Agency (U.S. EPA) to assess the impacts of electrical power generating plants using open-cycle, once-through cooling on the aquatic communities of the western basin of Lake Erie.

The particular objectives of the present study are: I) to estimate production of larval yellow perch in 1975-76 in U.S. waters of the western basin; II) to estimate natural mortality among larval yellow perch for the 20 to 30 day period following initiation of the pro-larval stage; III) to estimate the number of larval yellow perch entrained and killed in 1975-76 in cooling water of the 3100 megawatt Edison plant located at Monroe; IV) to estimate the percentage of total larval perch production in Michigan waters of the western basin that is lost in the cooling water cycle of the Monroe power plant; V) to estimate the percent loss in young-of-year recruitment attributable to entrainment mortality at the Monroe power plant; VI) to estimate the loss to the yellow perch fishery in western Lake Erie attributable to impingement and entrainment mortality occurring at the Monroe power plant. Impacts upon primary producers and benthic fauna have been previously reported (4) and are not discussed below.

Difficulties of Estimating the Effect of Water Intake Mortality Upon Larval Fish Survival

Yellow perch larvae enter U.S. waters of the western basin of Lake Erie from a variety of sources (Figure 1). Some larvae hatched in streams are carried into the coastal waters of the western basin by stream flow. Some are hatched in the Detroit River, Lake St. Clair, or along the Canadian shoreline and carried into Michigan waters by large scale basin water circulation (10). Larvae spawned in shoreline waters on the U.S. side (Figure 1) undoubtedly comprise the largest proportion of the total. The term "total production" is defined here as "all pro and post yellow perch larvae entering or hatched in U.S. waters of the western basin, including Maumee Bay, extending from the shoreline outward to the international boundary and eastward to the boundary of Ohio Zone E" (Figure 2). Thus, any larvae collected at sampling stations within the geographic boundary defined above are considered herein to have been produced in the U.S. waters of the western basin. This definition of total production allows valid comparisons between production and a) natural mortality of larvae, and b) numbers of larvae entrained by water intakes, since the process of entrainment and natural mortality are not

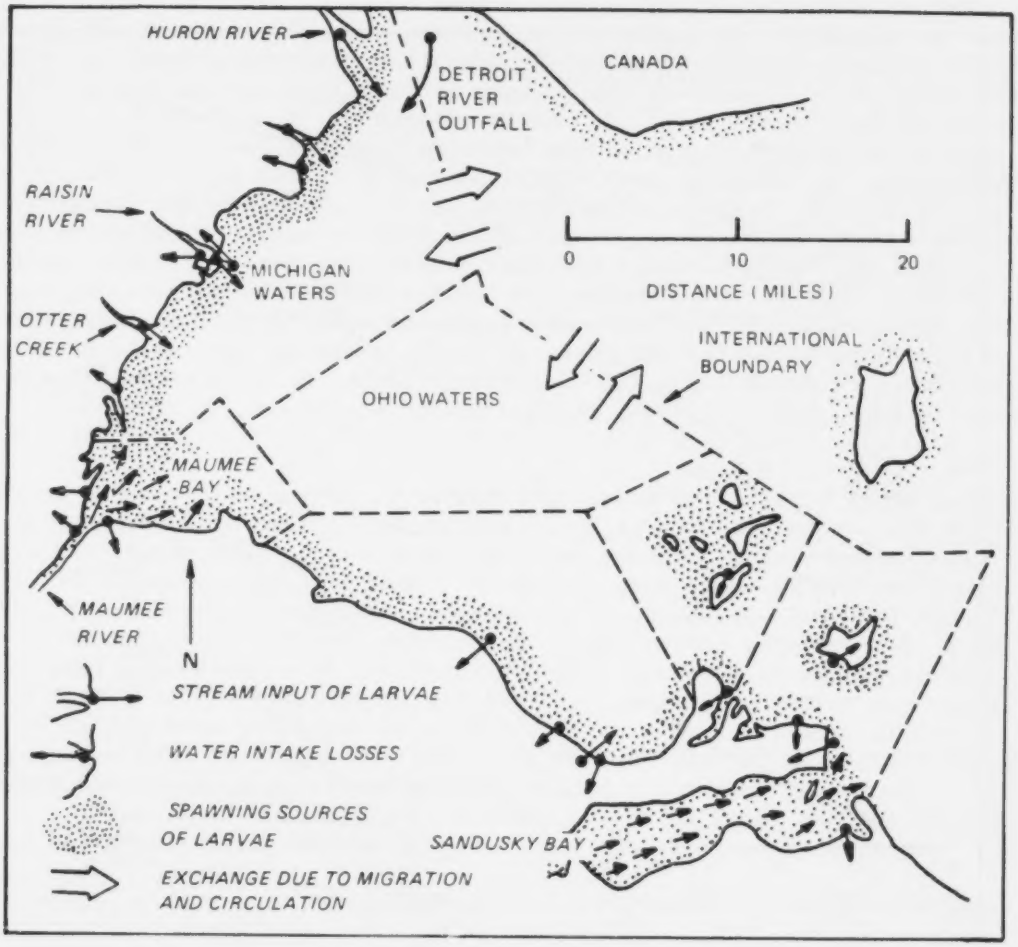

Figure 1. Western Lake Erie inputs and losses of yellow perch larvae.

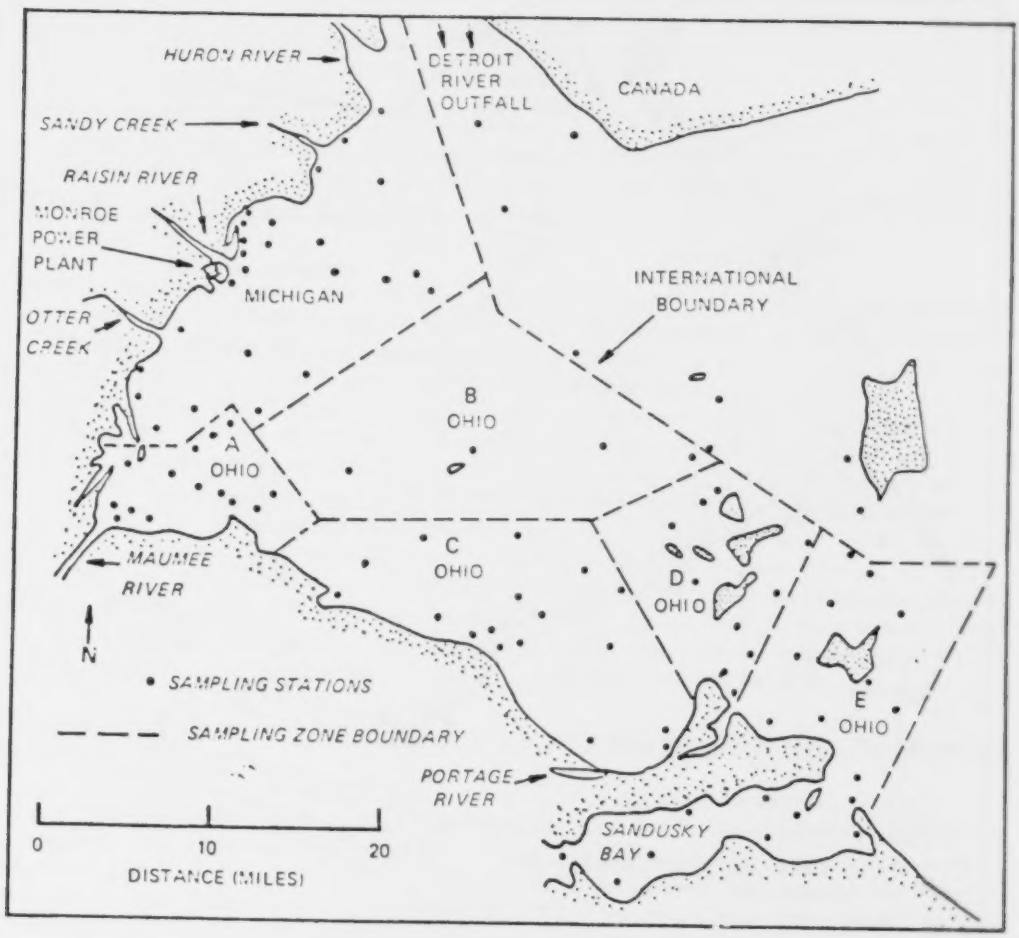

Figure 2. Western Lake Erie larval sampling stations in 1975-76.

functions of larval source of entry into the basin. Moreover, the above definition of production does not require independent estimates of larvae that enter the basin from streams or embayments external to it. If such estimates are, in fact, available then an estimate of the component of production due to basin spawners is possible.

If a direct approach is taken toward estimating production by basin spawners, the number of female spawners is multiplied by the number of larvae produced per female spawner. The resulting estimate of approximately $(7.0 \text{ to } 8.0)\times10^9$ larvae may be considered an upper limit to larval yellow perch production in U.S. waters of the western basin. An alternative approach is developed below for estimating production and natural mortality which utilizes measurements of larval densities rather than estimates of numbers of adult spawners and fecundity. Since abundance of larvae at any instant is the net integrative effect of production, water intake entrainment, natural mortality, migration, and recruitment into the young-of-year life stage, all of these factors are considered below. The method involves specification of a mathematical model that incorporates a parameter $\underline{h}$ of production and a parameter $\underline{p}$ of natural mortality, both of which are estimated numerically from field observations of larval densities. The model makes no assumption about joint behavior of production and natural mortality, i.e., the parameters $\underline{h}$ and $\underline{p}$.

Numerous possible sources of larvae sampling error exist. Perch larvae tend to move about in clumps, inhabiting beach areas, backwaters, and shallow embayments. As a result larvae may reach the young-of-year stage without becoming vulnerable to sampling gear. If this occurs, some clumps will never be sampled during a cruise; this contributes to an underestimate of abundance. The daytime distribution of perch larvae in the water column in the western basin is skewed with a high percentage clustered on or near bottom (Appendix A). Unless precautions are taken to sample the bottom concentrations of larvae both the mean and standard error of the estimate of mean concentration will be in error. Errors in the estimate of mean concentration propagate errors in estimates of production and natural mortality which, in turn, give rise to errors in the estimated percent of total production entrained in water intakes and recruitment into the young-of-year stage.

In addition to errors in estimates of the parameters $\underline{h}$ and $\underline{p}$ of production and natural mortality, modeling errors different from but leading to errors in parameter estimates may also occur. Modeling errors occur when incorrect assumptions are made about the mathematical representation of biotic or environmental processes which cause biased estimates of larval abundance and therefore, indirectly biased estimates of production and natural mortality. In summary, estimates of production and natural mortality of larval fishes can be in error due to four major causes shown in Diagram A.

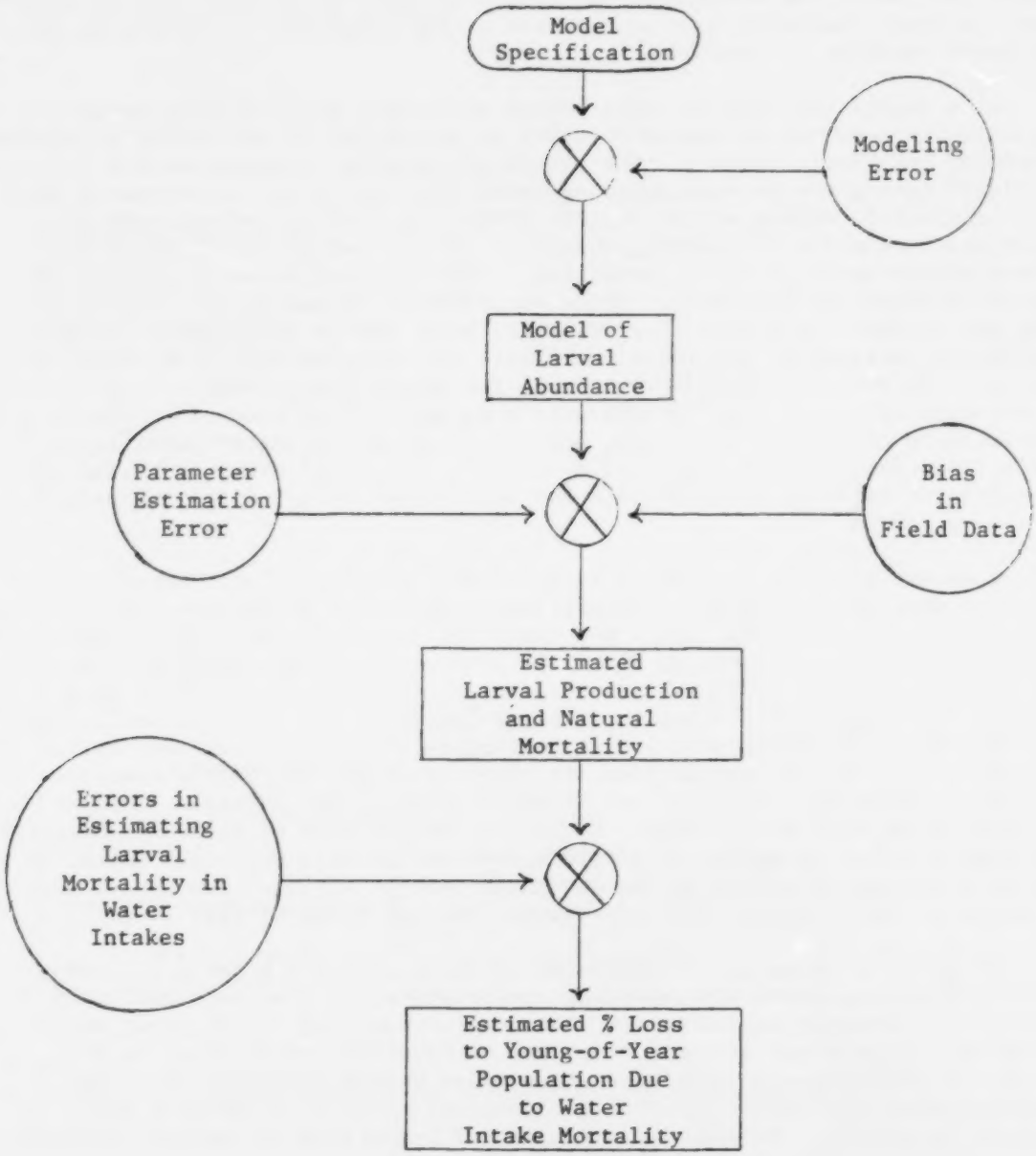
Model Specification
Modeling Error
Model of Larval Abundance
Parameter Estimation Error
Bias in Field Data
Estimated Larval Production and Natural Mortality
Errors in Estimating Larval Mortality in Water Intakes
Estimated % Loss to Young-of-Year Population Due to Water Intake Mortality

Diagram A: Sources of Error in Estimation Process

SECTION 3

DATA COLLECTION AND DISPLAY

Field surveys of standing crops of larval fishes provide the data base for estimates of production and natural mortality of larval yellow perch in 1975 and 1976. Estimates of the numbers of larval fishes entrained and killed in cooling water of the Edison plant at Monroe (4,6,7,9) provide the data base for estimating entrainment mortality and percentage of total annual production of larval yellow perch lost due to entrainment. Estimates of production and natural mortality of larval fishes are key requirements for an assessment of the impacts of specific point sources which produce larval mortality.

Data on larval perch concentrations shown in the graphs and tables below are based upon measurements taken at 68 stations in Ohio waters and at 20 stations in Michigan waters (Figure 2).[1] In addition, special sampling studies were conducted by the Michigan State University Institute of Water Resources. A complete listing of all species concentrations obtained at individual stations on specific cruises can be obtained from Cole (4), Hemmick, et al. (1), Herdendorf, et al. (5,6), and the Michigan Department of Natural Resources (2,3). Tables 1 through 11 summarize data in references (1), (2), (3), and (4) relative to yellow perch densities in Michigan waters (see also Figures 3-7). Water circulation in the western end of the basin is such that a large proportion of water from the Maumee estuary, driven by southwest winds, moves northeast into the Michigan zone from May to September whereas bottom waters from the Detroit River outfall move southwest along the bottom to replenish surface waters in the Michigan zone. Since larval densities measured at individual stations in 1975 were higher in the Maumee estuary and near the beaches between the Maumee estuary and the Raisin River than in waters north of the Raisin River, Michigan waters were tentatively subdivided into two surface zones. Analysis of 1976 data, however, did not show significant differences in mean concentrations between waters south of the Raisin River mouth and waters north of the mouth. Figure 3 compares concentrations of larval perch sampled in lake waters in the immediate vicinity of the river mouth with those in the upper discharge canal (4, Table B-26) of the Detroit Edison power plant. The lack of data on larval perch densities during May 1975 (Figure 7), earlier in their period of abundance, made it difficult to assess total production and percent of natural mortality of larval perch in

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[1] Depth zones in Michigan waters are reported in the English system of units to conform to the original data set. Calculations and results are reported in the metric system.

TABLE 1. OBSERVED DENSITIES OF LARVAL YELLOW PERCH IN MICHIGAN WATERS: 1975
Data Source: Ref. (2)

Depth Zone and Stations	Date		
	6/4 - 6/5	6/9 - 6/12	6/18 - 6/24
0'-6'			
18	0.64		0
19	0,3.19,3.19, 3.19,0.46		0,0,0,0,0
20	46.33		3.90
6'-12'			
1	0.42		7.13
4	0	0	6.20
7		0.53	0
11		0.42	0.35
14		0	0
			0
			6.06
			5.01
			4.98
12'-18'			1.57
8		0	0
12		0	0
15			1.11
17			0
			5.99
			2.10
			1.10
18'-24'			11.45
5		0.62	0
9		0.37	1.04
13		0.40	0
16		0.57	0
			0
			0
			0
24'-30'			0.72
6		0	0
10		0	0.31
			0.36,0,0,0
			0.68
			1.72,0.66,
			1.57,0.94,
			0.99

TABLE 1 (CONTINUED)

Depth Zone and Stations	Date			
	6/30 - 7/2	7/14 - 7/16	7/28 - 7/30	8/11 - 8/14
0'-6'				
18	0	0	0	0
19	0,0,0,0,0	0	0	0
20	0	0	0	0
6'-12'				
1	0	3.00	0	0
	0.33	0.33	0	0
4	0	0.36	0	0
	0	0.32	0	0
7	0	0	0	0
	0	0	0	0.35
11	0	0	0	0
	0	0	0	0
14	0	0	0	0
	0	0	0	0
12'-18'				
8	0	0	0	0
	0	0	0	0
12	0	0	0	0
		0	0	0
15	0	0	0	0
	0	0	0	0
17	0	0	0	0
	0	0	0.35	0
18'-24'				
5	0	0	0	0
	0	0	0	0
9	0	0	0	0
	0	0	0	0
13	0	0	0	0
		0	0	0
16	0	0	0	0
	0	0	0	0
24'-30'				
6	0	0.35	0	0
	0	1.22	0	0
10	0,0,0,0,0	0	0	0.38
	0	0	0	0

TABLE 1 (CONTINUED)

Depth Zone and Station	Date 9/2 - 9/5
0'-6'	
18	0
19	0
20	0
6'-12'	
1	0
	0
4	0
	0
7	0
	0
11	0
	0
14	0
	0
12'-18'	
5	0
	0
12	0
	0
15	0
	0
17	0
	0
18'-24'	
5	0
	0
9	0
	0
13	0
	0
16	0
	0
24'-30'	
6	0
	0
10	0
	0

Note: When two values are given for a single station on a given date, the lower and upper values are measurements at the bottom and top of the water column, respectively. If a single value is given, it is an average representing the entire water column, usually not more than three feet in depth.

TABLE 2. RESULTS OF NIGHT SAMPLING BY MSU ON MAY 21, 1975.
(# Perch Larvae/100 M^3)

Integrated Tow							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	19.2	11.0	8.4	7.2	10.1	11.1	4.63
Post L.	20.2	35.5	22.7	21.0	12.3	22.3	8.37

Surface							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	14.0	15.0	11.9	24.2	5.9	14.2	6.61
Post L.	10.4	18.8	7.9	10.4	26.6	14.8	7.77

Mid Depth							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	50.8	36.6	37.2	30.6	11.9	38.8	8.54
Post L.	45.2	30.5	14.3	22.3	8.9	24.2	14.29

Bottom							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	21.7	11.3	5.7	11.0	9.0	11.7	6.00
Post L.	24.8	5.7	0	8.2	6.0	8.9	9.37

TABLE 3. RESULTS OF NIGHT SAMPLING BY MSU ON MAY 22, 1975.
(# Perch Larvae/100 M^3)

	Integrated Tow						
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	7.0	9.5	7.9	2.1	9.7	7.2	3.09
Post L.	15.3	15.5	9.1	13.7	9.7	12.7	3.06

	Surface						
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	0	0	0	0	0	0	0

	Mid Depth						
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	2.9	0	0.6	1.3
Post L.	6.2	2.9	6.1	0	0	3.0	3.08

	Bottom						
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	13.6	17.6	9.2	20.3	2.8	12.7	6.94
Post L.	27.1	52.7	15.3	5.8	11.2	22.4	18.65

TABLE 4. RESULTS OF NIGHT SAMPLING BY MSU ON MAY 23, 1975.
(# Perch Larvae/100 M^3)

Integrated Tow							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	7.2	5.0	8.3	6.0	2.4	5.8	2.26
Post L.	8.4	23.6	15.4	25.0	14.4	17.4	6.90

Surface							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	3.2	0	3.3	0	1.3	1.78
Post L.	0	0	0	0	0	0	0

Mid Depth							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	5.8	0	5.8	5.5	5.4	4.5	2.52
Post L.	8.7	11.9	5.8	5.5	10.7	8.5	2.86

Bottom							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	5.9	6.5	6.4	10.9	5.8	7.1	2.15
Post L.	29.7	22.6	16.0	38.2	14.5	24.2	9.88

TABLE 5. RESULTS OF NIGHT SAMPLING BY MSU ON JUNE 16, 1975.
(# Perch Larvae/100 M^3)

Integrated Tow	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	8.8	8.6	17.9	1.0	12.1	9.7	6.14

Surface	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	2.9	0	2.8	0	0	1.1	1.56

Mid Depth	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	2.5	5.1	8.9	0	7.3	4.8	3.59

Bottom	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	9.5	17.4	4.9	14.2	10.1	11.2	4.78

TABLE 6. RESULTS OF NIGHT SAMPLING BY MSU ON JUNE 18, 1975.
(# Perch Larvae/100 M^3)

Integrated Tow							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	1	1	5.1	2.1	1.9	2.2	1.69

Surface							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	0	0	0	0	0	0	0

Mid Depth							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	0	0	0	15.1	0	3.0	6.75

Bottom							
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	2.4	7.3	9.3	17.2	15.9	10.4	6.15

TABLE 7. RESULTS OF NIGHT SAMPLING BY MSU ON JUNE 19, 1975.
(# Perch Larvae/100 M^3)

	Integrated Tow						
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	2.2	0	2.2	0	0	0.9	1.2

	Surface						
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	0	0	0	0	0	0	0

	Mid Depth						
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	0	0	0	0	0	0	0

	Bottom						
	Replication No.					Statistics	
	1	2	3	4	5	$\bar{x}$	s
Pro L.	0	0	0	0	0	0	0
Post L.	8.7	2.6	10.0	0	0	4.3	4.8

TABLE 8. SUMMARY OF NIGHT SAMPLING RESULTS BY MSU IN MAY-JUNE 1975
(# Perch Larvae/100 M^3)

Day	142	143	144	168	170	171
(1) Integ. Tow	33.48	19.90	23.14	9.68	2.22	0.88
(2) Surface (S)	29.00	0	1.30	1.14	0	0
(3) Mid Depth (M)	57.66	3.62	13.02	4.76	3.02	0
(4) Bottom (B)	20.68	35.12	31.30	11.22	10.42	4.26
(5) Avg. of S,M,B	35.78	12.91	15.21	5.71	4.48	1.42
(6) Avg. of (1)+(5)	34.63	16.41	19.18	7.70	3.35	1.15

TABLE 9. OBSERVED DENSITIES OF LARVAL YELLOW PERCH IN MICHIGAN WATERS: 1976.
Data Source: (3,4)

Depth Zone and MDNR Stations	4/13 MDNR		
	Prol.	EPL	LPL
0'-6'			
18	0	0	0
19	0	0	0
20	0	0	0
6'-12'			
1			
4			
7			
11			
14			
12'-18'			
8			
12			
15			
17			
18'-24'			
5			
9			
13			
16			
24'-30'			
6			
10			

Note: All concentrations are in units of #/100 M^3.

Note: For MDNR data: When two values are recorded at a single station and date, upper and lower values denote measurements at surface and bottom, respectively.

Note: No entry denotes no sample available.

Note: MSU samples taken only in general vicinity of MDNR stations.

TABLE 9 (CONTINUED)

Depth Zone and Stations	4/26-27 MDNR Prol	EPL	LPL	4/27 MSU Total Larvae	4/28 MDNR Prol	EPL	LPL	4/28 MSU Total Larvae
0'-6'								
18	0	0	0	586.5,49.7 36.1,12.9				11.7
19	0.911	0	0	3.7,23.6, 29.0,18.6				
20	0	0	0	5.0,51.8, 64.4				26.1
6'-12'								
1	0	0	0					
	1.18	0	0					
4					0.878	0	0	
					0.392	0.784	0	
7					1.32	0	0	
					2.74	0	0	
11				0.9				11.5
14				6.9,102.9, 142.9				130.1
12'-18'								
8				3.3	0.439	0	0	
					0.392	0	0	0
12					0	0	0	
				0	0.413	0	0	8.0
15								
17								
18'-24'								
5				1.7,0	0	0	0	
				1.0,0	0	0	0	5.0,1.0
9					0	0	0	
				1.0,4.1	0	0	0	0,0,0
13					0	0	0	
				0	0	0	0	2.1
16								
24'-30'								
6				1.0,0	0	0	0	0.9,1.0
10				0,0	0	0	0	1.1,0
				0.9,0				0

TABLE 9 (CONTINUED)

Depth Zone and Stations	4/29 MDNR Prol	EPL	LPL	4/29 MSU Total Larvae	5/14 MDNR Prol	EPL	LPL	5/14 MSU Total Larvae
0'-6'								
18					0	0	0	
19					0	2.73	0	
20				54.8				
6'-12'								
1								
4								
7								
11	1.53	0	0	4.8				
	9.01	0	0	9.2				
14	0.329	0	0					
	9.88	0	0					
12'-18'								
8				3.7				
12				2.1				
				0				
15	1.78	0	0					
	8.68	0	0					
17	0.909	0	0					
	9.59	0	0					
18'-24'								
5								
9								
13				0				
				0				
16	0.957	0	0					
	0	0	0					
24'-30'								
6								
10								

TABLE 9 (CONTINUED)

Depth Zone and Stations	5/16-18 MDNR Prol	EPL	LPL	5/16 MSU Total Larvae	5/18 MSU Total Larvae
0'-6'					
18				0	0
19					
20				2.2,0	1.1,0
6'-12'					
1	1.84	0	0		
	0	0	0		
4	0	0	0		
	0	0	0		
7				0	0
11					
14					
12'-18'					
8				1.9	0
12				0	
15					0
17					
18'-24'					
5				0.0,1.0	1.0,0,1.6
				4.7,3.2	0,3.0
9				0	
13				0	0
16					
24'-30'					
6					
10					
				4.1,0,2.2	3.0,0,1.2
				0,1.0,3.3	3.3,16.3,0
				15.4	1.1

TABLE 9 (CONTINUED)

Depth Zone and Stations	5/24 MDNR Prol	5/24 MDNR EPL	5/24 MDNR LPL	5/24 MSU Total Larvae	5/25-26 MDNR Prol	5/25-26 MDNR EPL	5/25-26 MDNR LPL	5/26 MSU Total Larvae
0'-6'								
18	0	47.7	0	3.0,1.0				1.1,2.1
19	0.456	1.37	0					
20	0	1.95	0	0,29.5, 7.3				0.8,3.4, 6.5,2.6
6'-12'								
1					0.303	0.303	0	
4					0.710	1.07	0	
7					0	0	0	
11					0	1.96	0	
14				0,0	0	0	3.44	
					0	1.60	0	
					0	0.878	0	0.9,2.8, 2.7
					0	1.57	0	
					0	0	5.24	1.9,1.0, 0,0
12'-18'				0,0,5.7	0	1.12	0	
8				2.0	0	0	0	3.0
12					0	0.784	0	
15				0,1.0	0	0.439	0	0
17					0	0	0.392	
					0	0	0.878	
					0	0.344	0	
					0	1.32	0	
18'-24'					0	0	0	
5				9.7				0
9					0	0	0	
13					0	1.96	0	
16				1.0,1.9	0	0.439	0	
				4.0	0	0.784	0	0,0,4.5
					0	0	0	
24'-30'					0	0	0	
6					0	12.3	0	
10					0	12.5	0	
					0	0	0	
					0	0	0	

TABLE 9 (CONTINUED)

Depth Zone and Stations	6/7 Prol	6/7 MDNR EPL	6/7 LPL	6/8 Prol	6/8 MDNR EPL	6/8 LPL
0'-6'						
18	0	0	0			
19	0	1.37	0			
20	0	0	4.39			
6'-12'						
1				0	0	0
				0	0.797	0
4				0	0	0
				0	0	0
7				0	0	0
				0	0	0
11				0	0	0.505
				0	0.459	0
14				0	0	0
				0	0	0
12'-18'						
8				0	0	0
				0	0	0
12				0	0	0
				0	0.358	0
15				0	0	0
				0	0	0
17				0	0	0
				0	0	0.437
18'-24'						
5				0	0	0
				0	0.392	0
9				0	0	0
				0	0.858	0
13				0	0.478	0
				0	0.08	0
16				0	0	0
				0	0	0
24'-30'						
6				0	0	0
				0	0	0
10				0	4.45	0
				0.909	0.454	0

TABLE 9 (CONTINUED)

Depth Zone and Stations	6/14 MSU Total Larvae	6/21 MSU Total Larvae	6/26 MSU Total Larvae
0'-6'			
	0,3.3	0,0,0,0,0	0,0
		0,0,0,0,0,0	
		0,0,0,0,0	
	0	0,0,0,2.0	0
6'-12'			
	0		0
12'-18'			
	0		0
	0		0
18'-24'			
	0		0
	0,0,0,0,0,0		0,0,0,0,0,0
24'-30'			
	0,0,14.4,12.4 0,0,0		0,0,0,0,0,0,0

TABLE 9 (CONTINUED)

Depth Zone and Stations	6/29 MDNR Prol	EPL	LPL
0'-6'			
18	0	0.636	0
19	0	0	0
20			0.488
6'-12'			
1			
4			
7			
11			
14			
12'-18'			
8			
12			
15			
17			
18'-24'			
5			
9			
13			
16			
24'-30'			
6			
10			

TABLE 9 (CONTINUED)

Depth Zone and Stations	7/16 Prol	7/16 MDNR EPL	7/16 LPL	7/9 Prol	7/9 MDNR EPL	7/9 LPL	7/19-20 Prol	7/19-20 MDNR EPL	7/19-20 LPL
0'-6'									
18	0	0	0				0	0	0
19	0	0	0				0,0,0	0,0,0	0,0,0
							0,0	0,0	0,0
20	0	0	0				0	0	0
6'-12'									
1	0	0	0.583	0	0	0.583	0	0.516	0
	0	0	0	0	0	0	0	0.967	0
4				0	0	0			
				0	0.392	0			
7				0	0	0			
				0	0.439	0			
11				0	0	0			
				0	0	0			
14				0	0	0			
				0	0	0			
12'-18'									
8				0	0	0			
				0	0.516	0			
12				0	0	0			
				0	0	0			
15				0	0	0			
				0	0	0			
17				0	0	0			
				0	0	0			
18'-24'									
5				0	0	0			
				0.350	0	0			
9				0	0	0			
				0	0	0			
13				0	0	0			
				0	0	0			
16				0	0	0			
				0	1.09	0			
24'-30'									
6				0	0.583	0			
				0.406	0	0			
10				0	0	0			
				0	0	0			

TABLE 9 (CONTINUED)

Depth Zone and Stations	7/21 Prol	7/21 MDNR EPL	7/21 LPL	7/28 Prol	7/28 MDNR EPL	7/28 LPL	8/3 Prol	8/3 MDNR EPL	8/3 LPL
0'-6'									
18				0	0	0			
19							0	0	0
20							0	0	0
6'-12'									
1									
4	0	0	0.334						
	0	0	0						
7	0	0	0				No yellow perch after 8/3.		
	0	0	0						
11				0	0	0			
				0	0	0			
14				0	0	0			
				0	0	0			
12'-18'									
8				0	0	0			
				0	0	0			
12				0	0	0.369			
				0	0	0			
15				0	0	0			
				0	0	0			
17				0,0,0	0,0,0	0,0,0			
				0,0	0,0	0,0			
				0,0,0	0,0,0	0,0,0			
				0,0	0,0	0,0			
18'-24'									
5	0	0	0						
	0	0	0						
9	0	0	0						
	0	0	0						
13				0	0	0			
				0	1.76	0			
16				0	0.388	0			
				0	0	0			
24'-30'									
6	0	0	0						
	0	1.70	0						
10	0	0	0						
	0	0	0						

TABLE 10. MEAN CONCENTRATIONS OF YELLOW PERCH IN MICHIGAN WATERS: 1976

Day	Mean	Standard Error (S.E.)	Mean ± S.E.
104	0	0	
118	5.63	1.94	3.69, 7.57
136	1.99	0.624	1.37, 2.61
145	2.14	0.56	1.58, 2.70
158	0.483	0.201	0.282, 0.684
188	0.145	0.057	0.088, 0.202
201	0.205	0.176	0.029, 0.381
209	0.112	0.078	0.034, 0.190

Note: Day 120 = 1 May.

Data Source: Table 9

Calculations given in Appendix C.

TABLE 11. ESTIMATED MEAN CONCENTRATION OF LARVAL PERCH IN MICHIGAN WATERS, 1976, BY STAGE OF DEVELOPMENT.

Date	Pro-Larvae	Early Post Larvae	Late Post Larvae	Total
104	0	0	0	0
118	5.61	0.02	0	5.63
136	0.80	1.19	0	1.99
145	0.04	2.06	0.04	2.14
158	0.041	0.386	0.056	0.483
188	0.033	0.103	0.009	0.145
201	0	0.202	0.003	0.205
209	0	0.095	0.017	0.112
215	0	0	0	0

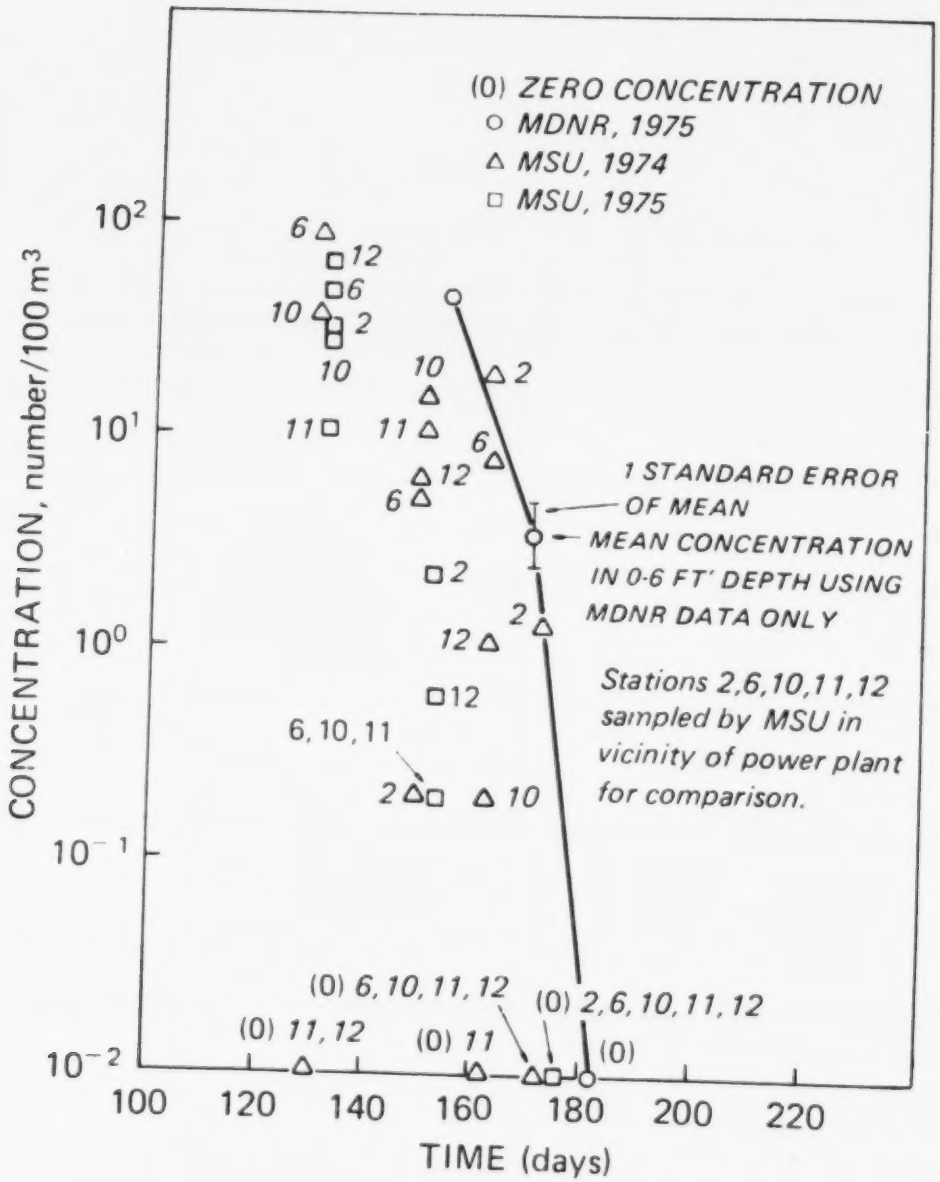
(0) ZERO CONCENTRATION
MDNR, 1975
MSU, 1974
MSU, 1975
1 STANDARD ERROR OF MEAN
MEAN CONCENTRATION IN 0-6 FT' DEPTH USING MDNR DATA ONLY
Stations 2,6,10,11,12 sampled by MSU in vicinity of power plant for comparison.
CONCENTRATION, number/100 m³
TIME (days)
(0) 11, 12
(0) 11
(0) 6, 10, 11, 12
(0) 2, 6, 10, 11, 12
(0)
6, 10, 11

Figure 3. Larval perch concentration in 0 to 6 ft. zone from Raisin River to Maumee Bay (1974-1975).

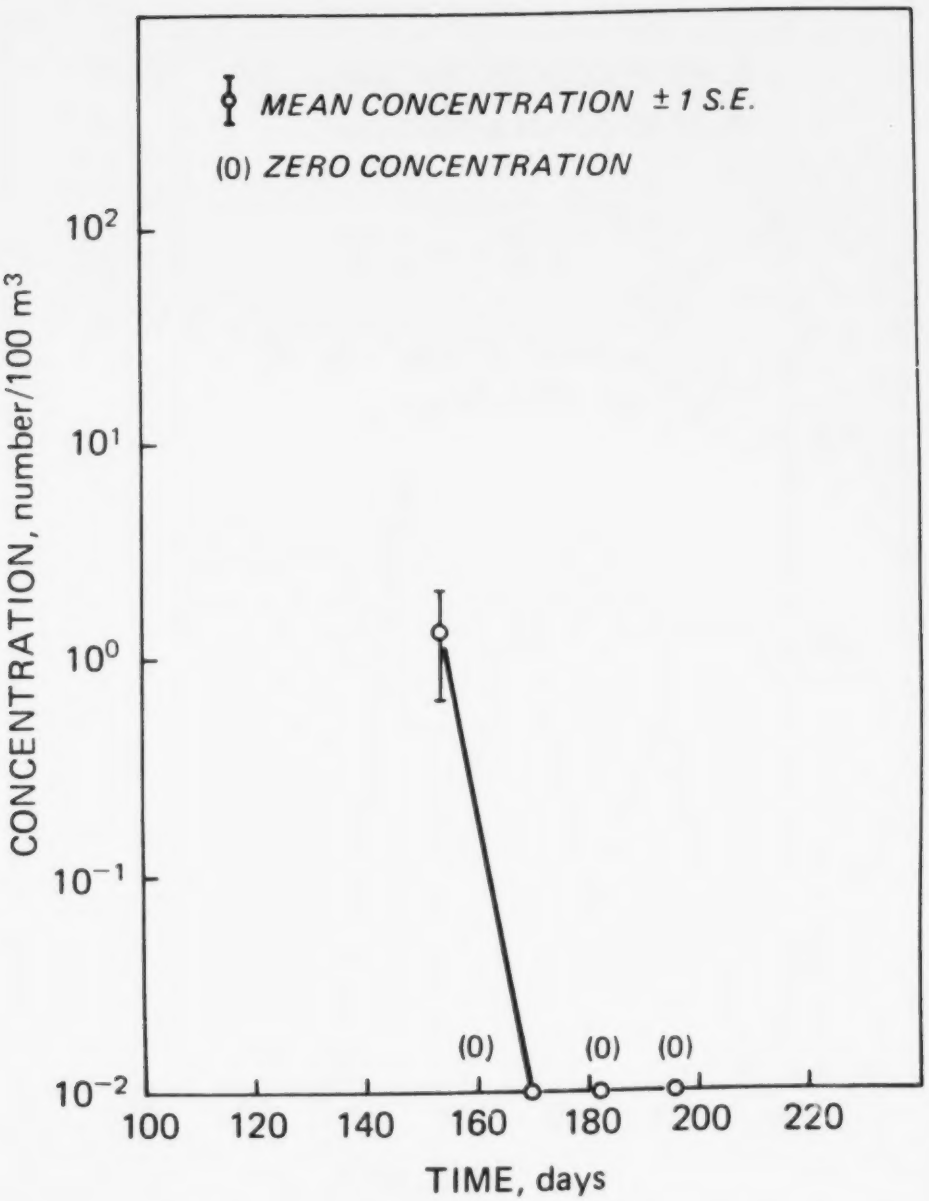

Figure 4. Larval perch concentration in 0 to 6 ft. zone from Raisin River to Maumee Bay (1975).
Data Source: Table 1.

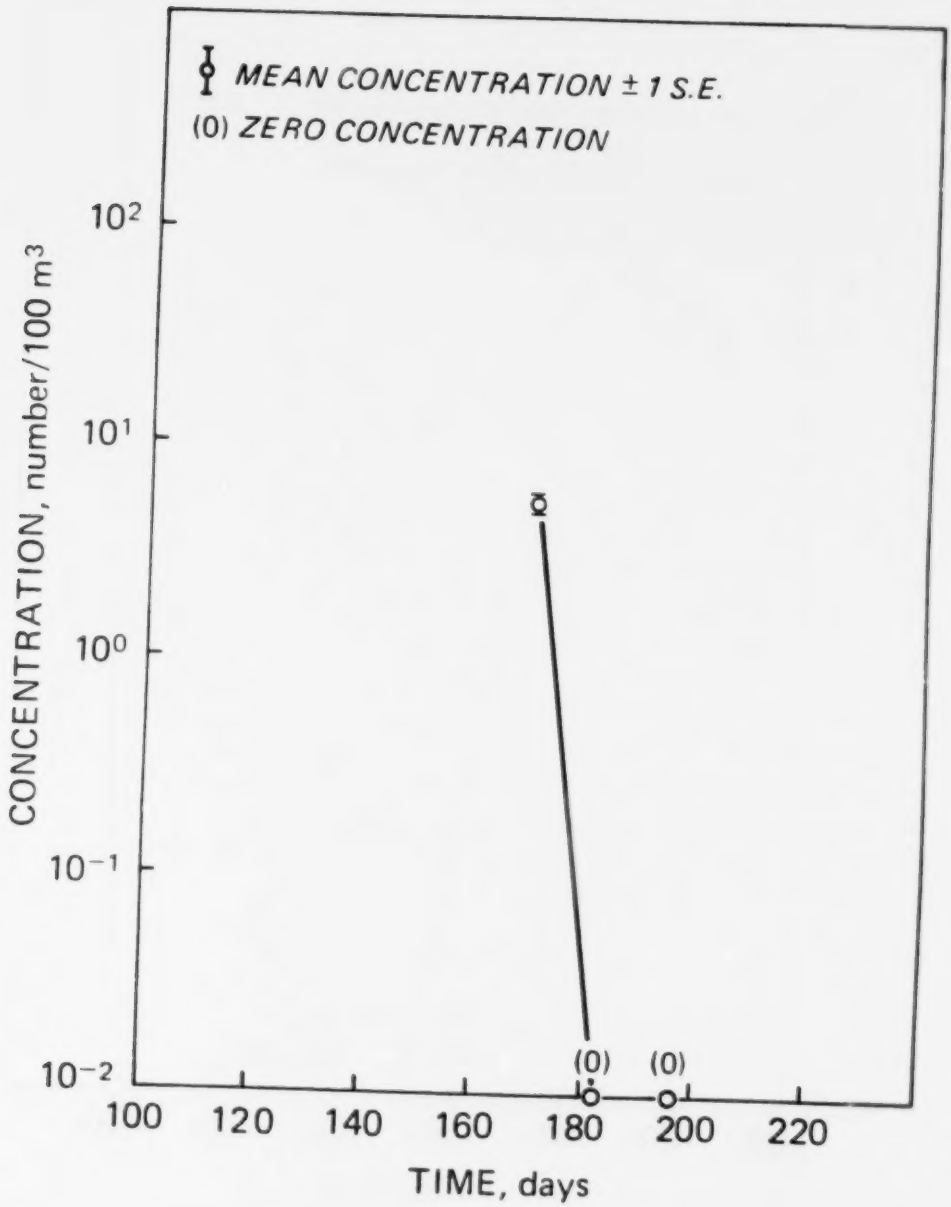

Figure 5. Larval perch concentration in 6 to 12 ft. zone from Raisin River to Maumee Bay (1975).
Data Source: Table 1.

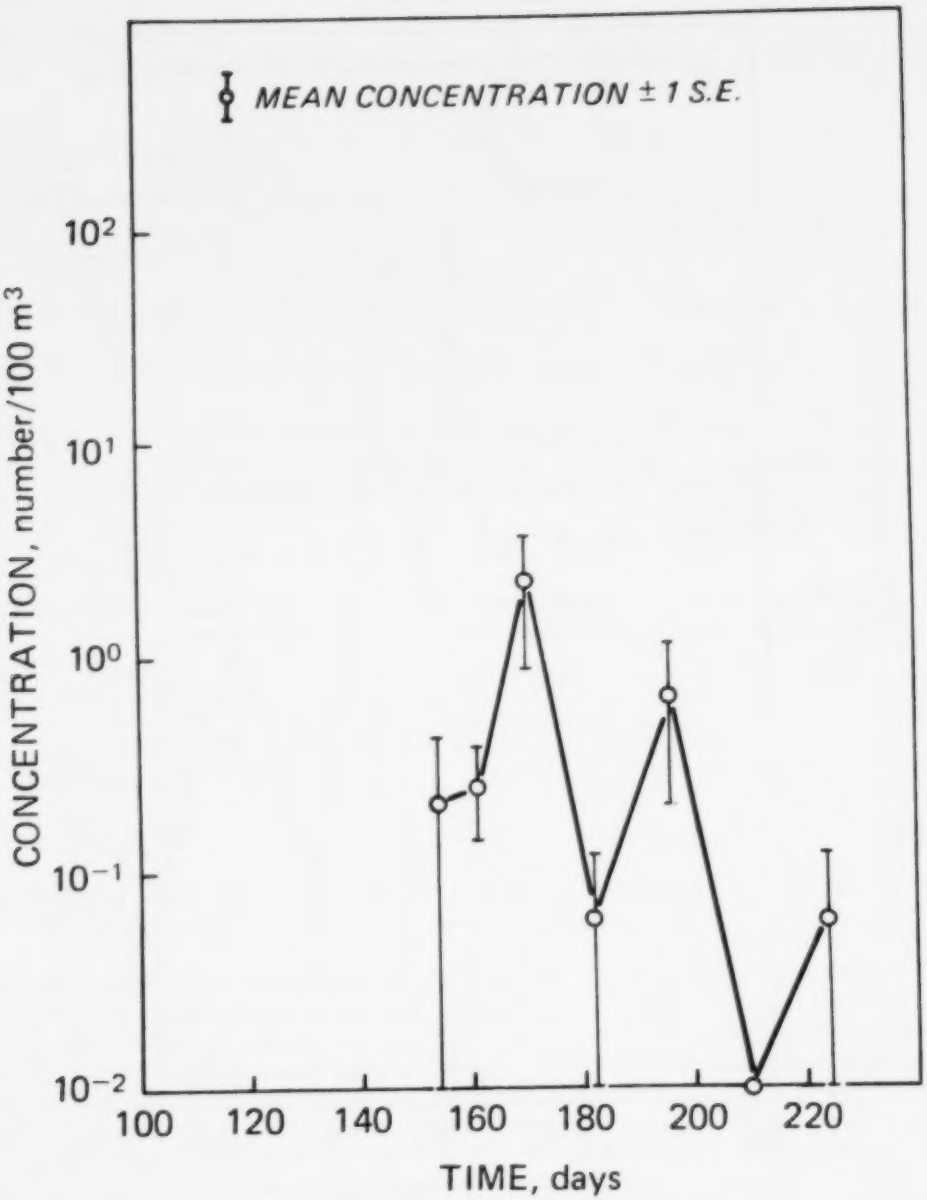

Figure 6. Larval perch concentration in 6 to 12 ft. zone from Raisin River to Huron River (1975).
Data Source: Table 1.

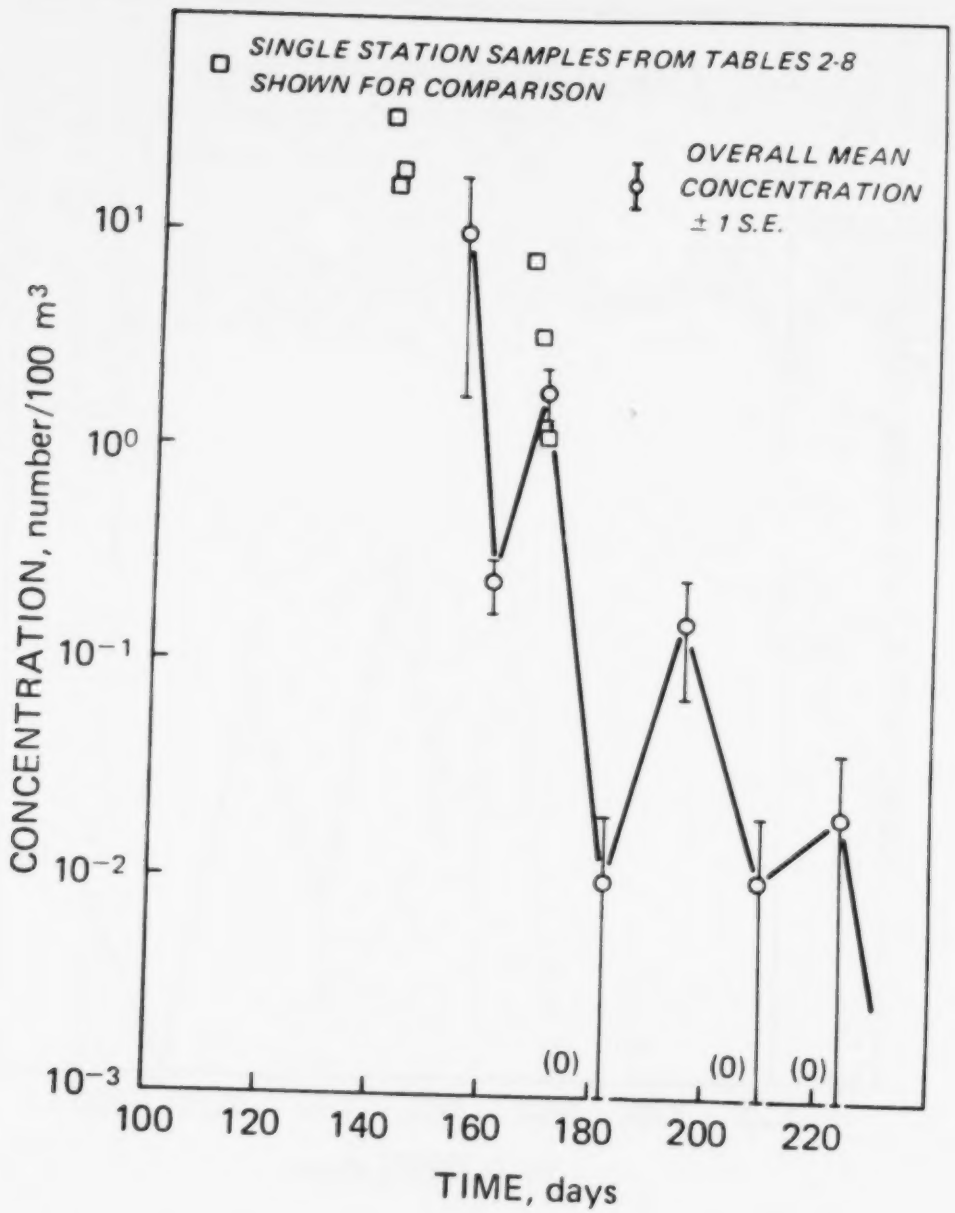

Figure 7. Mean larval perch concentration in Michigan waters (1975). Data Source: Table 1.

Michigan waters for that year. Mean larval concentrations shown in Figure 7 are obtained as weighted averages of concentrations sampled over all depth zones for which data are available on a given date:

$$\text{Mean Concentration on a sampling date} = \frac{1}{V_T} \quad (V_1 x_1 + \cdots + V_5 x_5) \tag{1}$$

where:

V_T = volume of Michigan waters = 4.976×10^8 M^3.

V_i = volume of i-th depth zone.

i = 1 corresponds to 0'-6' zone: 5.6×10^6 M^3

i = 2 corresponds to 6'-12' zone: 5.1×10^7 M^3

i = 3 corresponds to 12'-18' zone: 8.2×10^7 M^3

i = 4 corresponds to 18'-24' zone: 2.32×10^8 M^3

i = 5 corresponds to 24'-30' zone: 1.27×10^8 M^3

x_i = mean concentration in i-th depth zone averaged over all measurements obtained in that zone for the given sampling date.

$$\text{Standard Error of Mean} = \frac{1}{V_T} \left(V_1^2 \frac{s_1^2}{n_1} + \cdots + V_5^2 \frac{s_5^2}{n_5}\right)^{1/2} \tag{2}$$

where:

s_i = standard deviation of all n_i measurements obtained in i-th depth zone on the given sampling date.

Sample concentrations obtained in each depth zone are aggregated for purposes of calculating mean concentrations (Figure 7). Also plotted in Figure 7 are sample concentrations obtained at night. The latter samples were collected in the 6' to 12' depth zone approximately 1 kilometer offshore from the mouth of the Raisin River (Tables 2-8). Densities of larval yellow perch obtained at night were found to be higher than those obtained during the day and probable causes are discussed by Cole (4). A subsequent statistical analysis of day to night differences (Appendix A) showed that they were significant ($P < .10$ for surface and $P < .005$ for bottom concentrations), indicating that estimates of yellow perch larval abundance or production based upon densities observed only during daylight hours are low.

Larval perch densities measured in Michigan waters in 1976 are listed in Table 9 and plotted in Figures 9 through 14. Concentrations are highest in the 0' to 6' depth early in the spawning period. A 1976 overall mean concentration for Michigan waters is calculated and shown in Table 10 and Figure

Figure 8. Locations of MSU sampling stations in vicinity of Monroe Power Plant.
Data Source: Ref.(4).

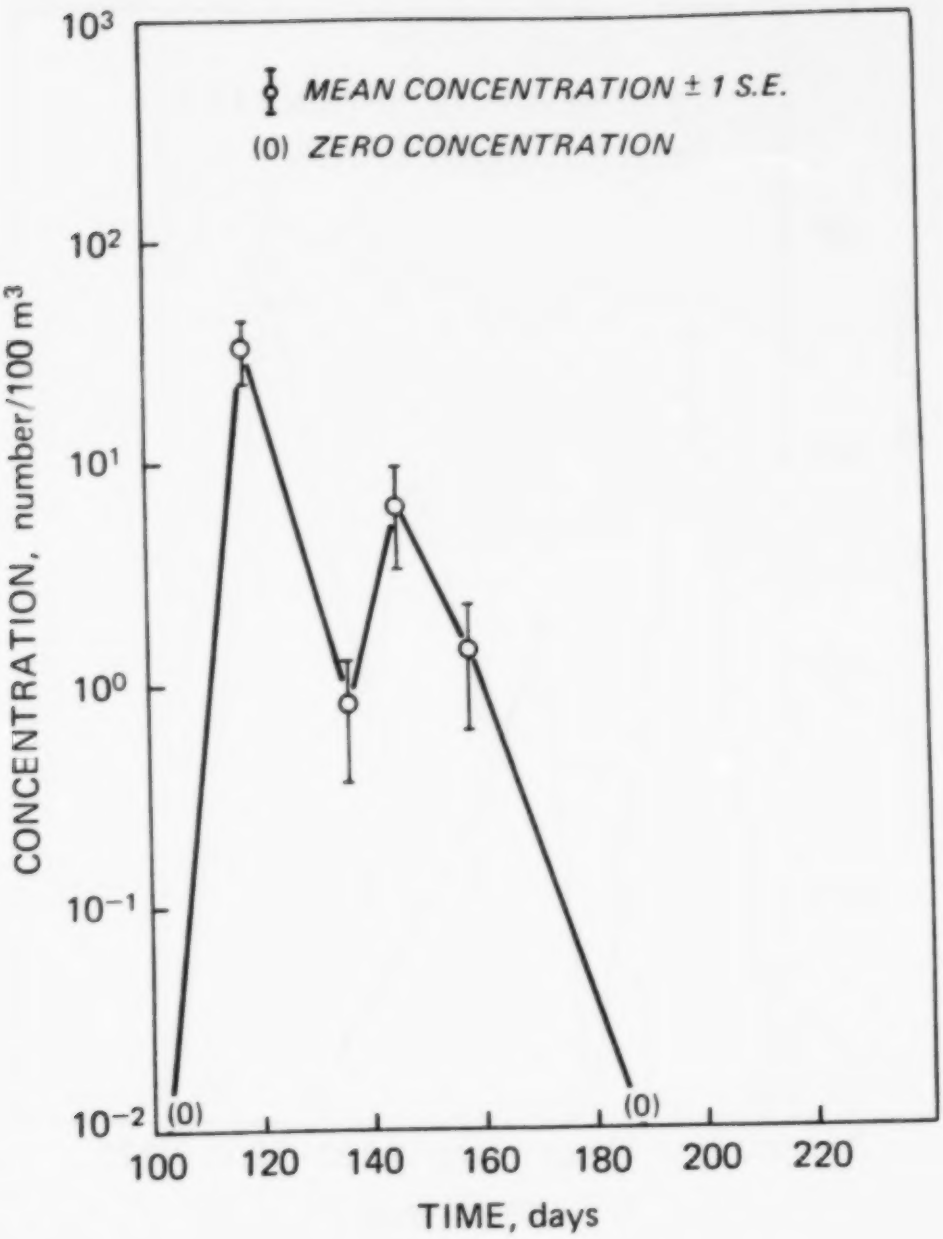

Figure 9. Larval perch concentration in 0 to 6 ft. zone from Raisin River to Maumee Bay (1976).
Data Source: Table 9.

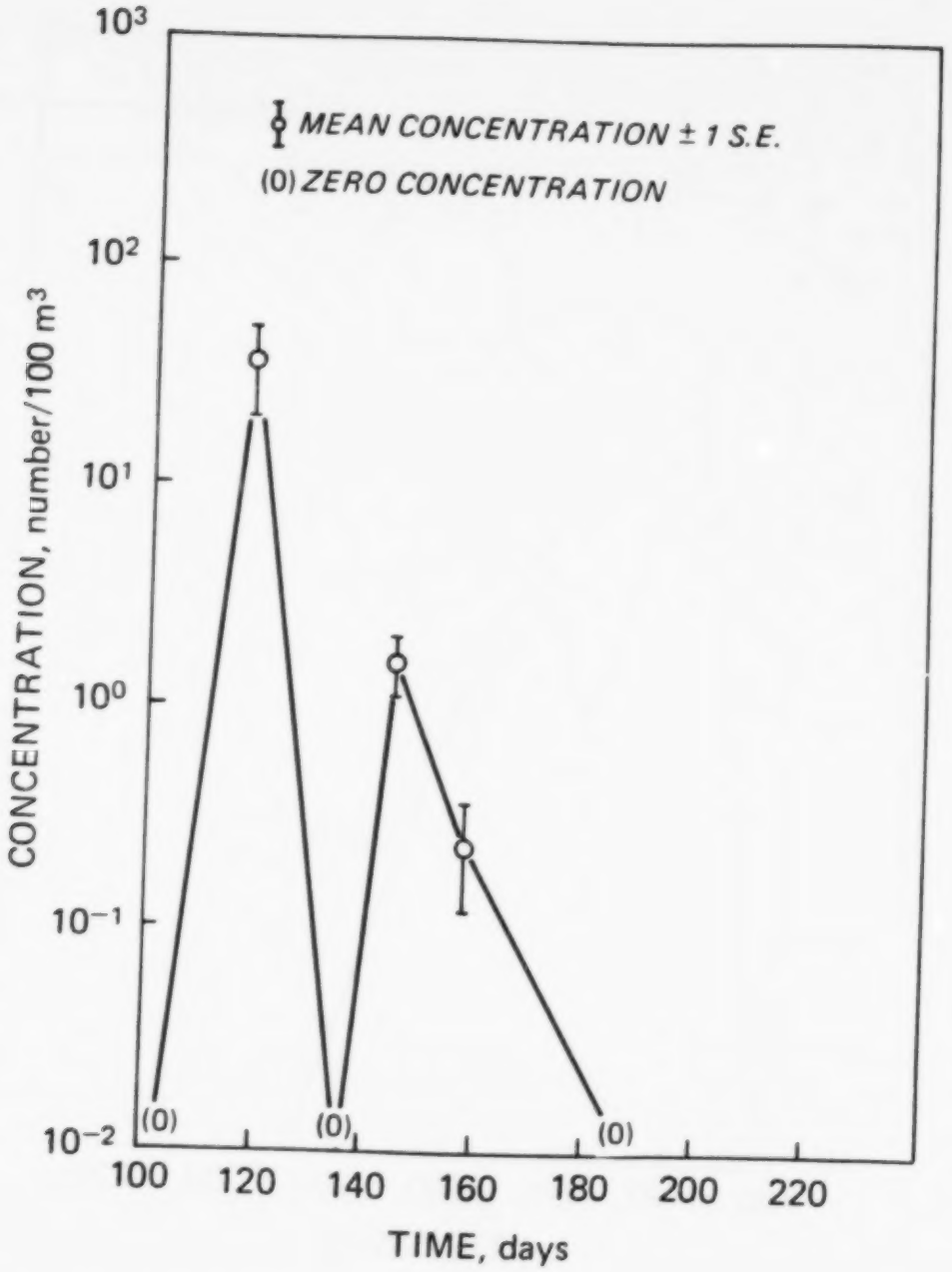

Figure 10. Larval perch concentration in 6 to 12 ft. zone from Raisin River to Maumee Bay (1976).
Data Source: Table 9.

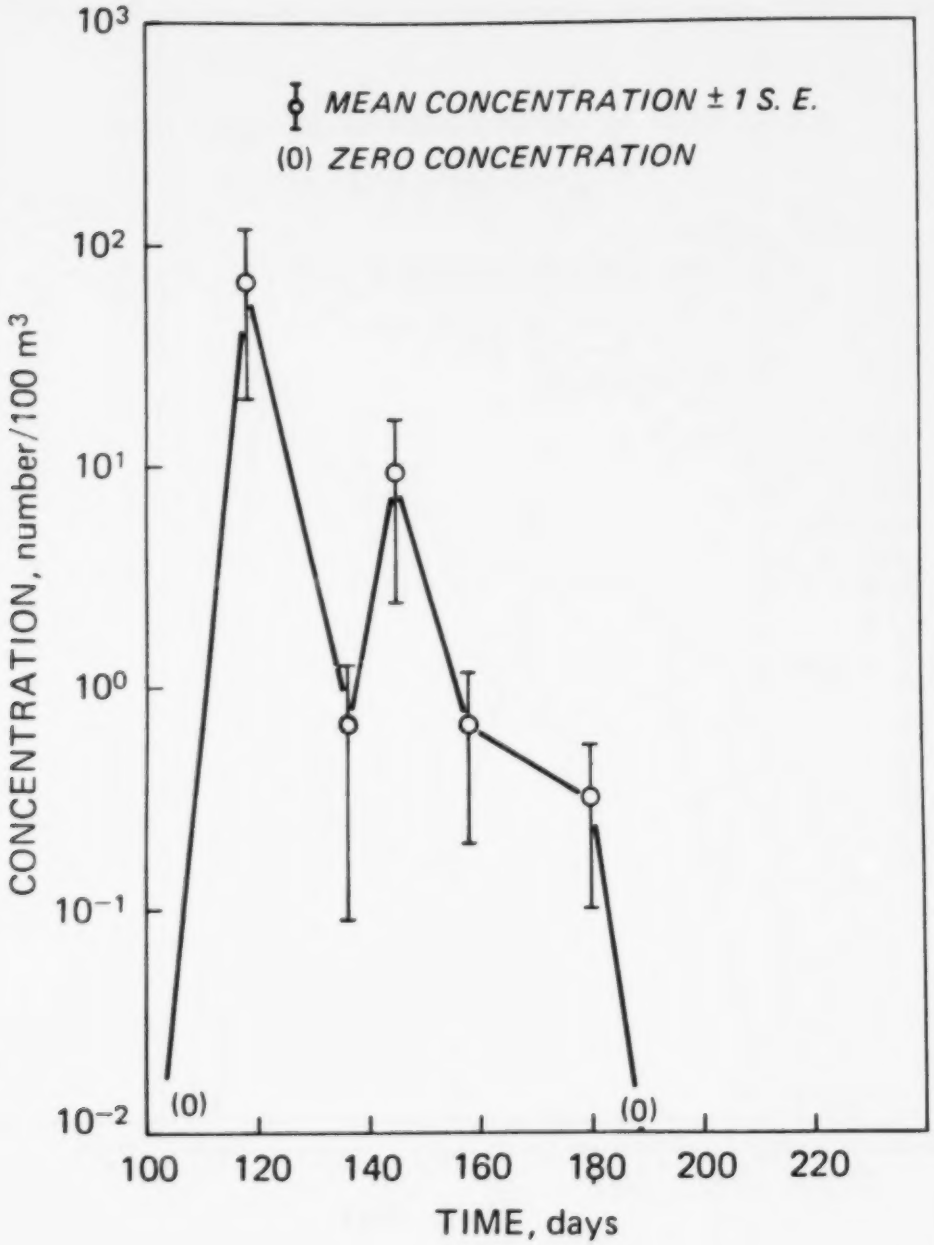

Figure 11. Larval perch concentration in 0 to 6 ft. zone from Raisin River to Huron River (1976).
Data Source: Table 9.

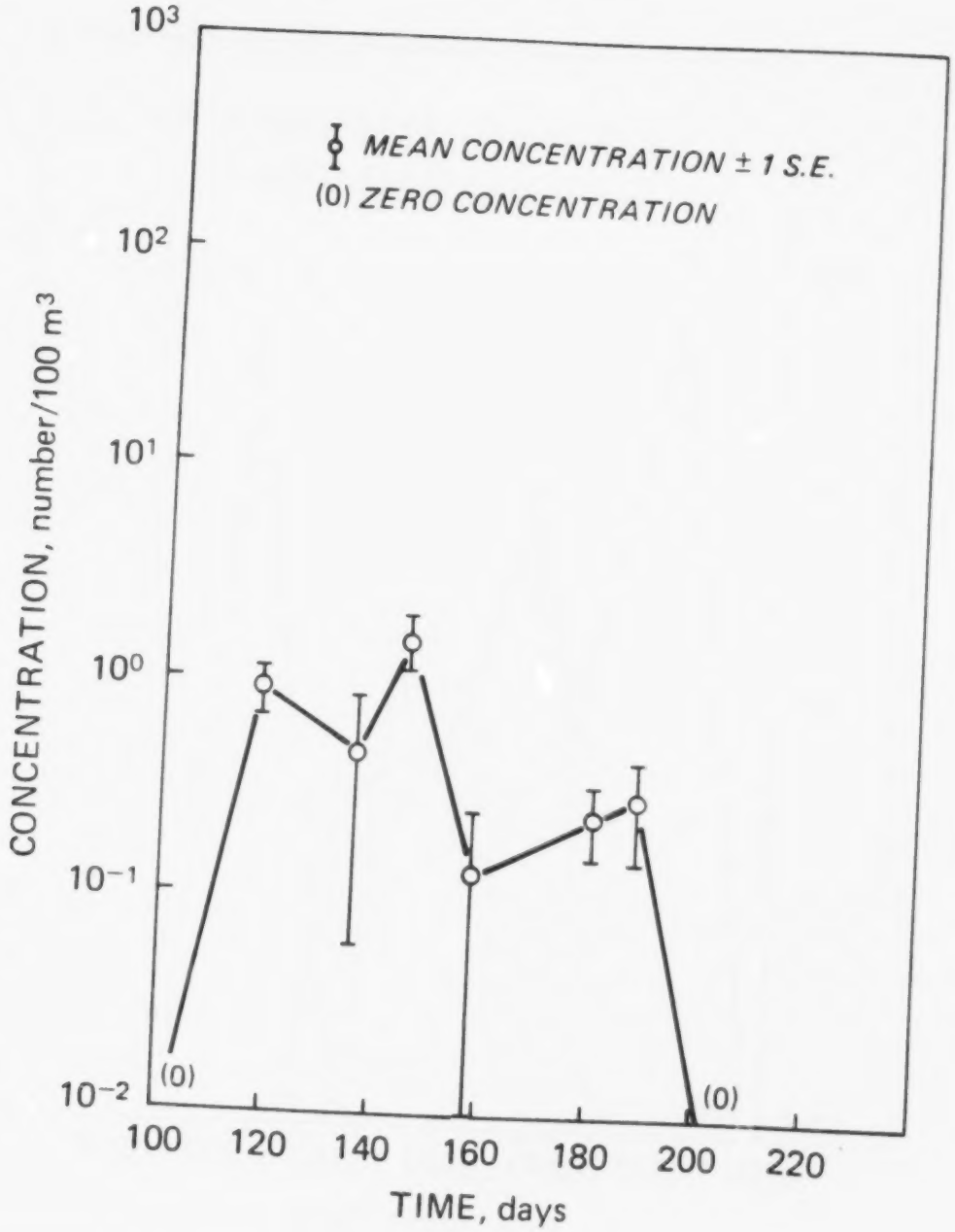

Figure 12. Larval perch concentration in 6 to 12 ft. zone from Raisin River to Huron River (1976).
Data Source: Table 9.

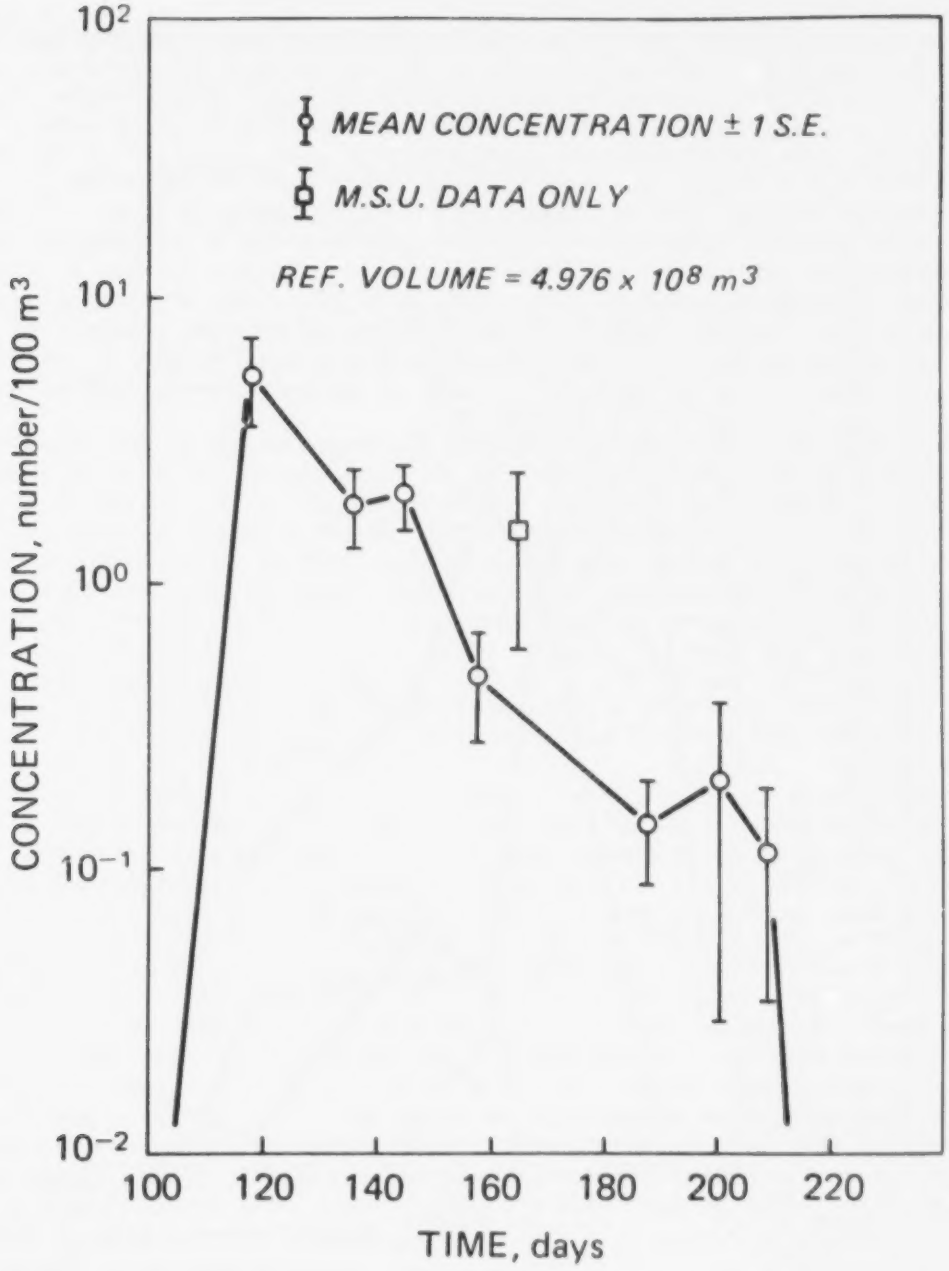

Figure 13. Mean larval perch concentration in Michigan waters (1976). Data Source: Table 9.

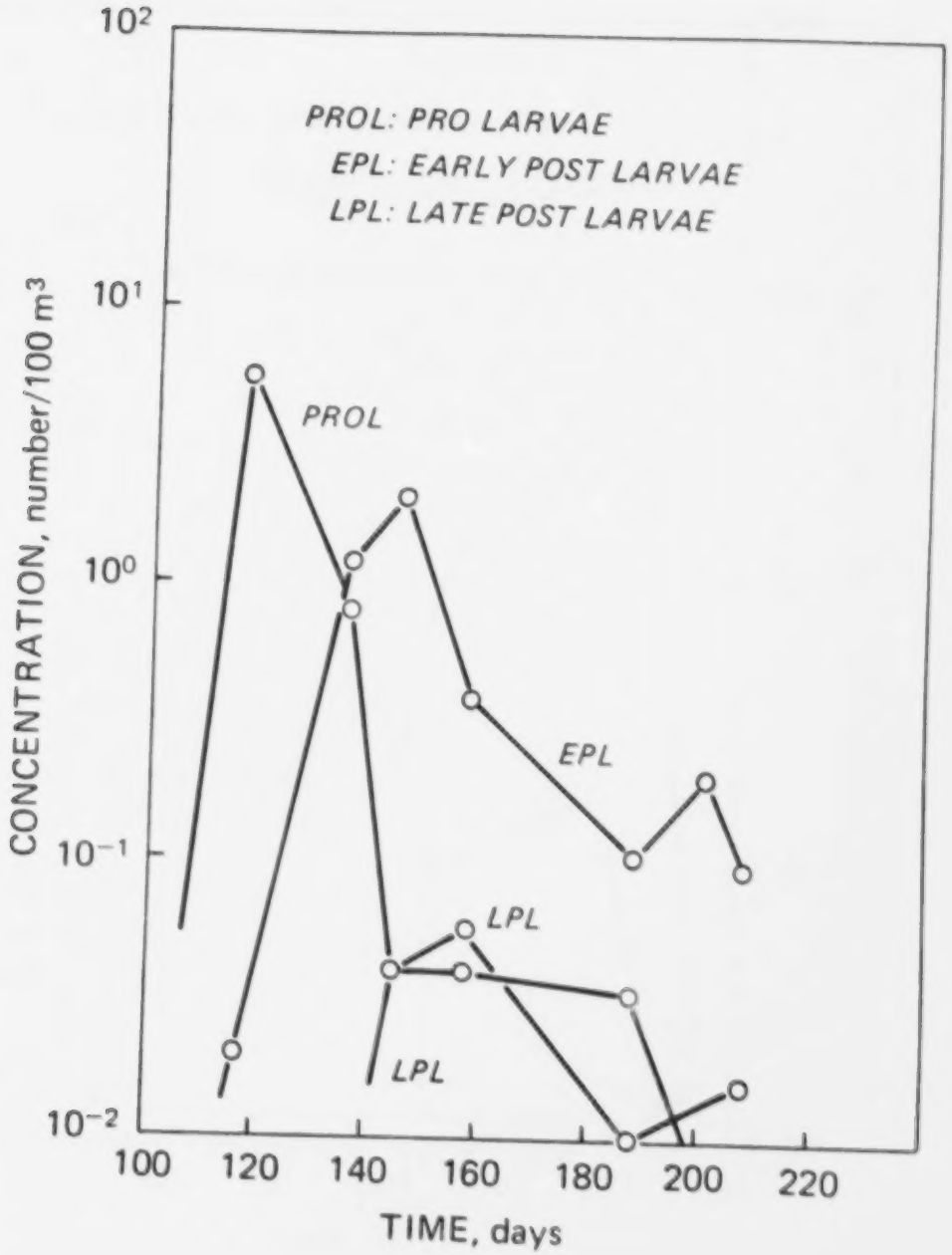

Figure 14. Mean larval perch concentration in Michigan waters (1976) by stage of maturation.
Data Source: Table 9.

13. Before an overall mean concentration for Michigan waters was calculated, it was determined that observed differences in mean concentrations by depth zone were statistically significant. Tests of significance for differences (Appendix B) by depth zone for Michigan waters in 1976 showed that concentrations in the 0' to 12' zone were significantly higher ($P < .025$) during the period of observed peak abundance than concentrations measured in other depth zones. Furthermore, statistical analysis showed that 0' to 12' and 12' to 30' zones could be aggregated to compute mean concentrations and standard errors. Calculations of mean and standard errors for Figure 13 are shown in Appendix C. Figure 13 presents a typical picture of the temporal variation in larval abundance: a rapid buildup occurs due to a high production rate followed by a decline due to natural mortality, migration, and net avoidance. As larvae increase in age to 20 to 30 days, they become progressively more capable of avoiding capture by sampling gear, so that eventually no larvae are observed in samples (see also Figures 15 and 16 for similar patterns occurring in Ohio waters in 1975 to 76).

Concentrations shown in Figure 13 on any given date represent the sum of pro-larvae, early post larvae, and late post larvae. A disaggregation of these data corresponding to the three stages of larval development (for each sampling date) is plotted in Figure 14. Approximately five to seven days elapse before pro-larvae develop into an early post larval stage and approximately 10 additional days elapse before the late post larval stage. For present purposes, yellow perch larvae are considered to be recruited into the young-of-year stage after 25 days of life. Figure 14 shows that larval production began approximately on day 102 (April 12, 1976) and continued at a relatively high rate until approximately day 140, a period of about five weeks. Abundance tapered off, finally terminating between days 190 and 200.[1]

For 1975 and 1976, mean concentrations of larval perch in Ohio waters of the western basin exhibited temporal variations similar to those shown in Michigan waters (Figures 15 and 16). The mean values shown in Figures 15 and 16 are weighted averages of concentrations in Zones A,B,C,D, and E. The temporal patterns of abundance are similar for both years, although peak production occurred approximately three weeks earlier in 1976 and was possibly lower in 1976 than in 1975. Means and standard errors are calculated by following equations (1) and (2) and using Tables 12A-12E. In the 1976 plot standard errors on each date are calculated by pooling estimates of mean concentrations obtained in Zones A through E. Figures 18 to 30 show estimated mean concentrations in the 0 to 2 meter and 2 to 4 meter depth zones for sectors A, C, and D for 1975-76. The plots do not clearly show which year produced the highest larval abundance. Even when all depth zones are accounted for (Figures 15 and 16) the picture remains somewhat clouded but it is indicated that perch larvae were less abundant in 1976 than in 1975, based upon comparison of mean concentrations.

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[1]In order to incorporate observations obtained by the MSU Institute of Water Research (Table 9) into Figure 14 it is assumed that the proportions of larvae in each developmental stage obtained from analysis of MDNR observations holds as well for MSUIWR observations.

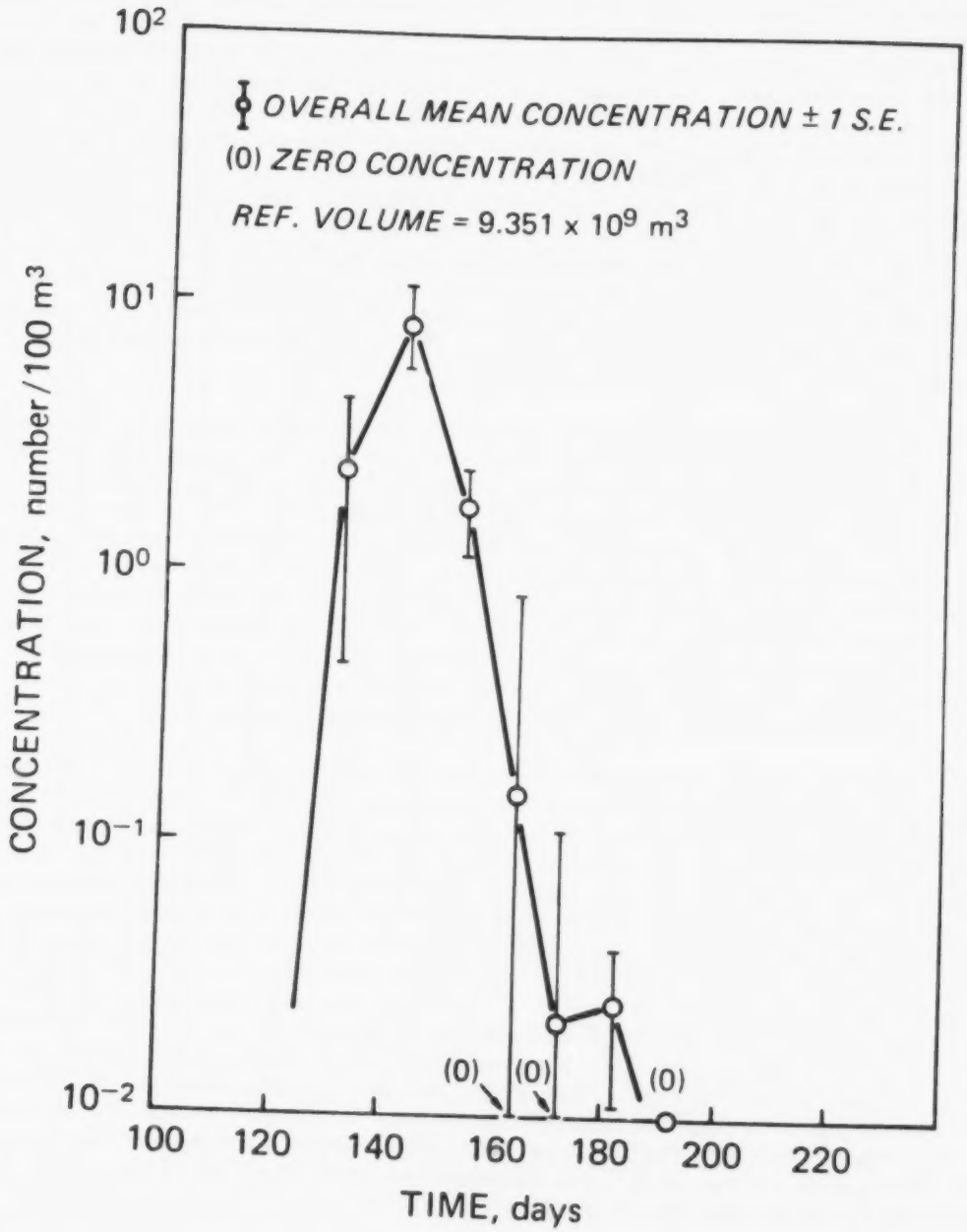

Figure 15. Mean larval perch concentration in Ohio waters (1975, Zones A-E).
Data Source: Tables 12A and 12B.

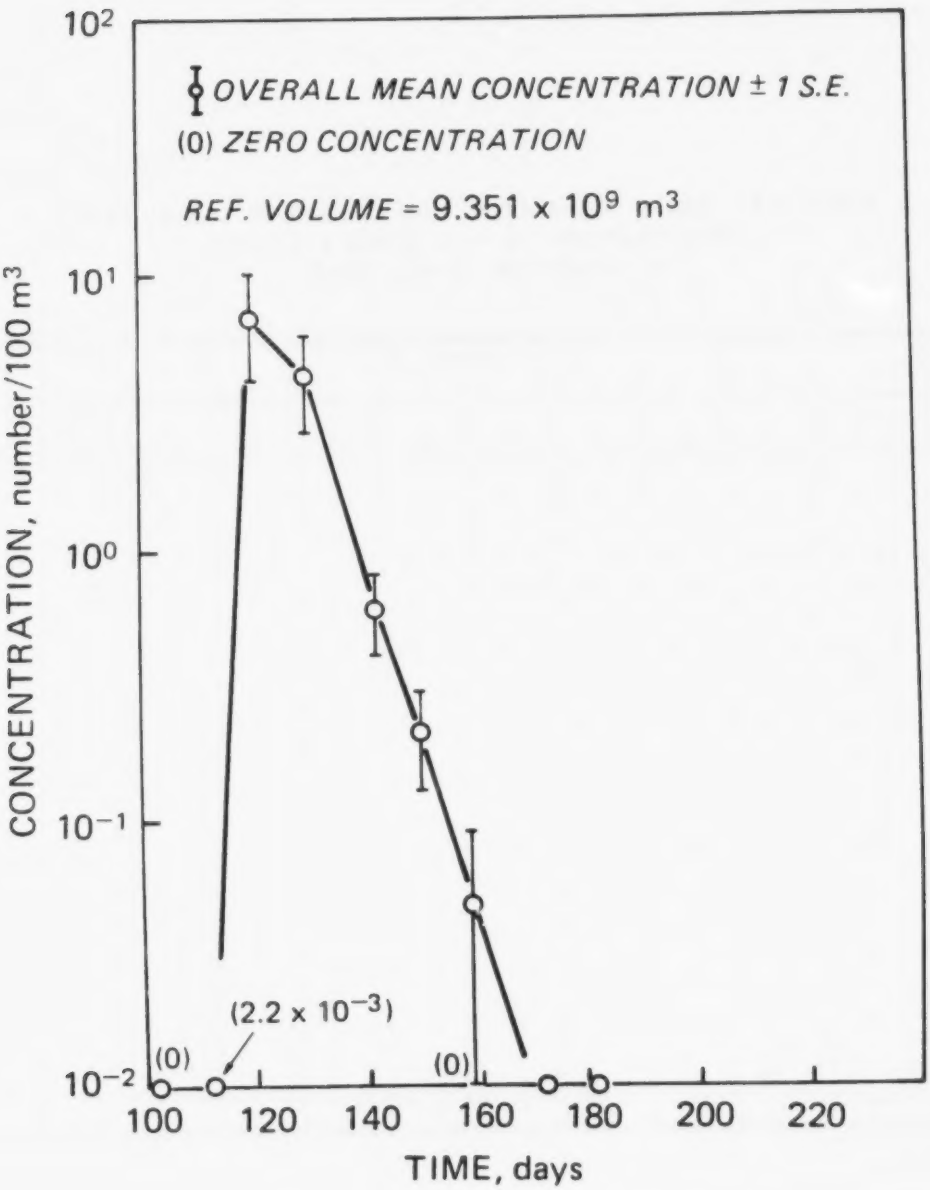

Figure 16. Mean larval perch concentration in Ohio waters (1976, Zones A-E).
Data Source: Tables 12C and 12D.

TABLE 12A. MEANS AND STANDARD DEVIATIONS OF LARVAL PERCH CONCENTRATIONS IN OHIO ZONES A-E, 1975.
Data Source: Reference (5)

Sector A Time	Depth Zone 1		2		3		4		5		6	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
May 12-14	-	-	-	-	-	-	-	-	-	-	-	-
May 22-25	15.75 n=8	13.08	7.00 n=4	8.72	5.00 n=4	3.56	-	-	-	-	-	-
June 1-4	12.12 n=8	8.1	4.75 n=4	4.65	1.25 n=4	0.96	-	-	-	-	-	-
June 11-17	18.37 n=8	51.57	1.50 n=4	3.00	0 n=4	0	-	-	-	-	-	-
June 21-23	2.75 n=8	3.99	0 n=4	0	0 n=4	0	-	-	-	-	-	-
July 1-3	0	0	0	0	0	0	-	-	-	-	-	-
July 11-15	0	0	0	0	0	0	-	-	-	-	-	-
Aug. 1-4	0	0	0	0	0	0	-	-	-	-	-	-
Aug. 27-Sept. 8	0	0	0	0	0	0	-	-	-	-	-	-

TABLE 12A (CONTINUED)

Sector B	Depth Zone 1		2		3		4		5		6	
Time	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
May 12-14	-	-	-	-	-	-	-	-	-	-	-	-
May 22-25	-	-	-	-	-	-	1.25 n=4	2.5	0.50 n=4	0.58	1.50 n=4	3.0
June 1-4	-	-	-	-	-	-	0 n=2	0	0 n=2	0	1 n=2	1.41
June 11-17	-	-	-	-	-	-	0	0	0	0	0	0
June 21-23	-	-	-	-	-	-	0	0	0	0	0	0
July 1-3	-	-	-	-	-	-	0 n=2	0	0.25 n=4	0.50	0 n=2	0
July 11-15	-	-	-	-	-	-	0	0	0	0	0	0
Aug. 1-4	-	-	-	-	-	-	0	0	0	0	0	0
Aug. 27-Sept. 3	-	-	-	-	-	-	0	0	0	0	0	0

TABLE 12A (CONTINUED)

Sector C Time	Depth Zone 1		2		3		4		5		6	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
May 12-14	49.5 n=10	156.53	78.5 n=6	139.3	0.5 n=6	1.22	0 n=4	0	0 n=4	0	-	-
May 22-25	614.12 n=8	1782	72.12 n=8	115.0	3.33 n=6	6.74	4.5 n-4	4.8	10.5 n=4	17.8	-	-
June 1-4	2.94 n=8	8.11	1.94 n=8	1.77	2.0 n=6	3.63	0.5 n=4	0.58	0.25 n=4	0.50	-	-
June 11-17	0 n=8	0	0.31 n=8	0.37	0 n=6	0	0 n=4	0	0 n=4	0	-	-
June 21-23	0.37 n=8	0.98	0.17 n=8	0.11	0 n=3	0	0 n=2	0	0 n=2	0	-	-
July 1-3	0	0	0	0	0	0	0	0	0	0	-	-
July 11-15	0	0	0	0	0	0	0	0	0	0	-	-
Aug. 1-4	0	0	0	0	0	0	0	0	0	0	-	-
Aug. 27-Sept. 3	0	0	0	0	0	0	0	0	0	0	-	-

TABLE 12A (CONTINUED)

Sector D Time	Depth Zone 1		2		3		4		5		6	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
May 12-14	0.25 n=4	0.5	6.0 n=4	12.0	0 n=4	0	0.75 n=4	1.5	0 n=2	0	0 n=2	0
May 22-25	5.0 n=4	10.0	0.87 n=4	1.5	12.25 n=4	14.2	6.0 n=4	7.7	5.5 n=4	4.1	3.75 n=4	7.5
June 1-4	1.25 n=4	1.26	0.25 n=4	0.50	17.25 n=4	21.47	0.75 n=4	0.96	1.75 n=4	1.71	2.0 n=4	2.45
June 11-17	0	0	0	0	0	0	0	0	0.12 n=4	0.25	0	0
June 21-23	0	0	0	0	0	0	0	0	0	0	0	0
July 1-3	0	0	0	0	0	0	0	0	0	0	0	0
July 11-15	0	0	0	0	0	0	0	0	0	0	0	0
Aug. 1-4	0	0	0	0	0	0	0	0	0	0	0	0
Aug. 27-Sept. 3	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 12A (CONTINUED)

Sector E Time	Depth Zone 1		2		3		4		5		6	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
May 12-14	7.75 n=4	15.5	0.33 n=6	0.82	0.75 n=4	1.5	0.75 n=4	1.5	0	0	0	0
May 22-25	5.75 n=4	10.84	11.92 n=6	26.47	13.5 n=4	17.6	13.75 n=4	16.3	5.25 n=4	5.5	9.75 n=4	14.93
June 1-4	0.5 n=4	1.0	0.25 n=4	0.5	1.0 n=4	0.82	1.0 n=2	0	1.5 n=4	3.0	4.5 n=4	6.45
June 11-17	0	0	0.25 n=4	0.29	0	0	0	0	0.12 n=4	0.25	0	0
June 21-23	0	0	0	0	0	0	0	0	0	0	0	0
July 1-3	0	0	0	0	0	0	0	0	0	0	0	0
July 11-15	0	0	0	0	0	0	0	0	0	0	0	0
Aug. 1-4	0	0	0	0	0	0	0	0	0	0	0	0
Aug. 27-Sept. 3	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 12B. ESTIMATED ABUNDANCE OF LARVAL PERCH IN OHIO ZONES A-E, 1975.

Date	Sector A	B	C	D	E
5/12-14	1.09×10^7*	0**	2.05×10^8	2.32×10^6	3.47×10^6
5/22-25	2.41×10^7	3.05×10^7	4.14×10^8	9.24×10^7	2.21×10^8
6/1-4	1.35×10^7	1.06×10^7	2.30×10^7	3.98×10^7	7.72×10^7
6/11-17	1.19×10^7	0	7.56×10^5	9.95×10^5	5.02×10^5
6/21-23	1.58×10^6	2.42×10^6	4.95×10^5	0	0
7/1-3	0	0	0	0	0
7/11-15	0	0	0	0	0
8/1-4	0	0	0	0	0
8/27 to 9/8	0	0	0	0	0

*: Estimated by using average concentrations in Zones C, D and E.
**: Zone B not sampled on 5/12-14.

TABLE 12C. ESTIMATED MEAN CONCENTRATION IN OHIO WATERS, 1975 (ZONES A-E).

Date	Mean Concentration (No. per 100 M^3)	Standard Error (S.E.)	Mean ± S.E.
5/12-14	2.38	1.93	$(4.5\times10^{-1}, 4.31)$
5/22-25	8.36	2.74	(5.62, 11.10)
6/1-4	1.75	6.3×10^{-1}	(1.12, 2.38)
6/11-17	1.51×10^{-1}	6.6×10^{-1}	$(0, 8.0\times10^{-1})$
6/21-23	2.22×10^{-2}	8.7×10^{-2}	$(0, 1.1\times10^{-1})$
7/1-3	2.60×10^{-2}	1.5×10^{-2}	$(1.1\times10^{-2}, 4.1\times10^{-2})$
7/11-15	0	0	
8/1-4	0	0	
8/27 to 9/8	0	0	

TABLE 12D. ESTIMATED ABUNDANCE OF LARVAL PERCH IN OHIO ZONES A-E, 1976 (BY DEPTH ZONE).
Data Source: Reference (6)

Sector A / Depth Zone / Date	1	2	3	4	5	6
Apr. 12-16	0	0	0	0	0	0
Apr. 21-23 (Partial)	0	0	0	0	0	0
Apr. 28 - May 1	6.9×10^6	4.9×10^6	3.9×10^6	0	0	0
May 8-11	1.3×10^6	9.3×10^5	2.6×10^6	0	0	0
May 20-23	7.9×10^5	0	4.3×10^5	0	0	0
May 30 (Partial)	-	-	-	-	-	-
June 7-9	0	7.0×10^5	0	0	0	0
June 19-25	0	0	0	0	0	0
June 30-July 7	0	0	0	0	0	0

TABLE 12D (CONTINUED)

Sector B / Depth Zone / Date	1	2	3	4	5	6
Apr. 12-16				0	0	0
Apr. 21-23 (Partial)				-	-	-
Apr. 28 - May 1				0	0	0
May 8-11				4.0×10^6	2.4×10^6	1.6×10^7
May 20-23				0	0	0
May 30 (Partial)				-	-	-
June 7-9				3.9×10^6	0	0
June 19-25				0	0	0
June 30-July 7				0	0	0

TABLE 12D (CONTINUED)

Sector C \ Depth Zone / Date	1	2	3	4	5	6
Apr. 12-16	0	0	0	0	0	
Apr. 21-23 (Partial)	5.4×10^4	0	0	0	0	
Apr. 28 - May 1	3.0×10^6	2.6×10^8	2.6×10^8	5.7×10^7	2.4×10^6	
May 8-11	7.9×10^5	1.2×10^6	1.1×10^6	7.6×10^6	1.2×10^6	
May 20-23	4.3×10^5	4.3×10^6	6.3×10^6	1.9×10^6	3.6×10^6	
May 30 (Partial)	-	-	-	-	-	
June 7-9	0	0	0	0	0	
June 19-25	0	0	0	0	0	
June 30-July 7	0	0	0	0	0	

TABLE 12D (CONTINUED)

Sector D / Depth Zone / Date	1	2	3	4	5	6
Apr. 12-16	0	0	0	0	0	0
Apr. 21-23 (Partial)	-	-	-	-	-	-
Apr. 28 - May 1	2.3×10^6	6.8×10^6	3.1×10^5	1.7×10^7	1.5×10^7	8.0×10^6
May 8-11	5.9×10^5	5.3×10^6	2.8×10^6	8.7×10^6	8.9×10^7	3.5×10^7
May 20-23	1.8×10^5	1.8×10^5	6.1×10^5	1.4×10^7	1.0×10^7	0
May 30 (Partial)	0	6.0×10^4	4.6×10^6	0	0	1.6×10^6
June 7-9	0	6.0×10^4	0	0	0	0
June 19-25	0	0	0	0	0	0
June 30-July 7	0	0	0	0	0	0

TABLE 12D (CONTINUED)

Sector E \ Depth Zone / Date	1	2	3	4	5	6
Apr. 12-16	0	0	0	0	0	0
Apr. 21-23 (Partial)	-	-	-	-	-	-
Apr. 28 - May 1	0	0	5.5×10^6	6.9×10^7	0	0
May 8-11	6.0×10^5	1.5×10^6	3.9×10^7	2.3×10^7	5.8×10^7	1.3×10^8
May 20-23	3.5×10^4	1.0×10^6	3.5×10^6	3.3×10^6	3.6×10^6	3.8×10^6
May 30 (Partial)	0	6.6×10^5	0	0	1.8×10^6	0
June 7-9	0	0	0	0	0	0
June 19-25	0	0	0	0	0	0
June 30-July 7	0	0	0	0	0	0

TABLE 12E. ESTIMATED ABUNDANCE OF LARVAL PERCH IN OHIO ZONES A-E, 1976.

Date	Sector A	B	C	D	E
4/12-16	0	0	0	0	0
4/21-23	0	-	5.4×10^4	-	-
4/28 to 5/1	1.57×10^7	0	5.83×10^8	4.94×10^7	7.45×10^7
5/8-11	4.83×10^6	2.24×10^7	1.19×10^7	1.41×10^8	2.52×10^8
5/20-23	1.22×10^6	0	1.65×10^7	2.50×10^7	1.52×10^7
5/30	-	-	-	6.26×10^6	2.46×10^6
6/7-9	7.0×10^5	3.9×10^6	0	6.0×10^4	0
6/19-25	0	0	0	0	0
6/30 to 7/7	0	0	0	0	0

TABLE 12F. ESTIMATED MEAN CONCENTRATION IN OHIO WATERS, 1976 (ZONES A-E).

Date	Total Abundance	Mean Concentration (No. per 100 M^3)	Estimated Standard Error*	Mean $\pm$ 1 Standard Error
4/12-16	0	0		
4/21-23	5.4×10^4	2.2×10^{-3}	1.32×10^{-3}	$(8.8 \times 10^{-4}, 3.5 \times 10^{-3})$
5/28 to 5/1	7.23×10^8	7.72	3.31	(4.41, 11.0)
5/8-11	4.32×10^8	4.62	1.79	(2.83, 6.41)
5/20-23	5.79×10^7	6.19×10^{-1}	2.04×10^{-1}	$(4.1 \times 10^{-1}, 8.2 \times 10^{-1})$
5/30	8.72×10^6	2.13×10^{-1}	8.42×10^{-2}	$(1.3 \times 10^{-1}, 2.97 \times 10^{-1})$
6/7-9	4.66×10^6	4.98×10^{-2}	4.04×10^{-2}	$(9.4 \times 10^{-3}, 9.0 \times 10^{-2})$
6/19-25	0	0	0	
6/30 to 7/7	0	0	0	

*Standard Error estimated from mean concentration across sectors on a given date, except for 4/28 to 5/1.

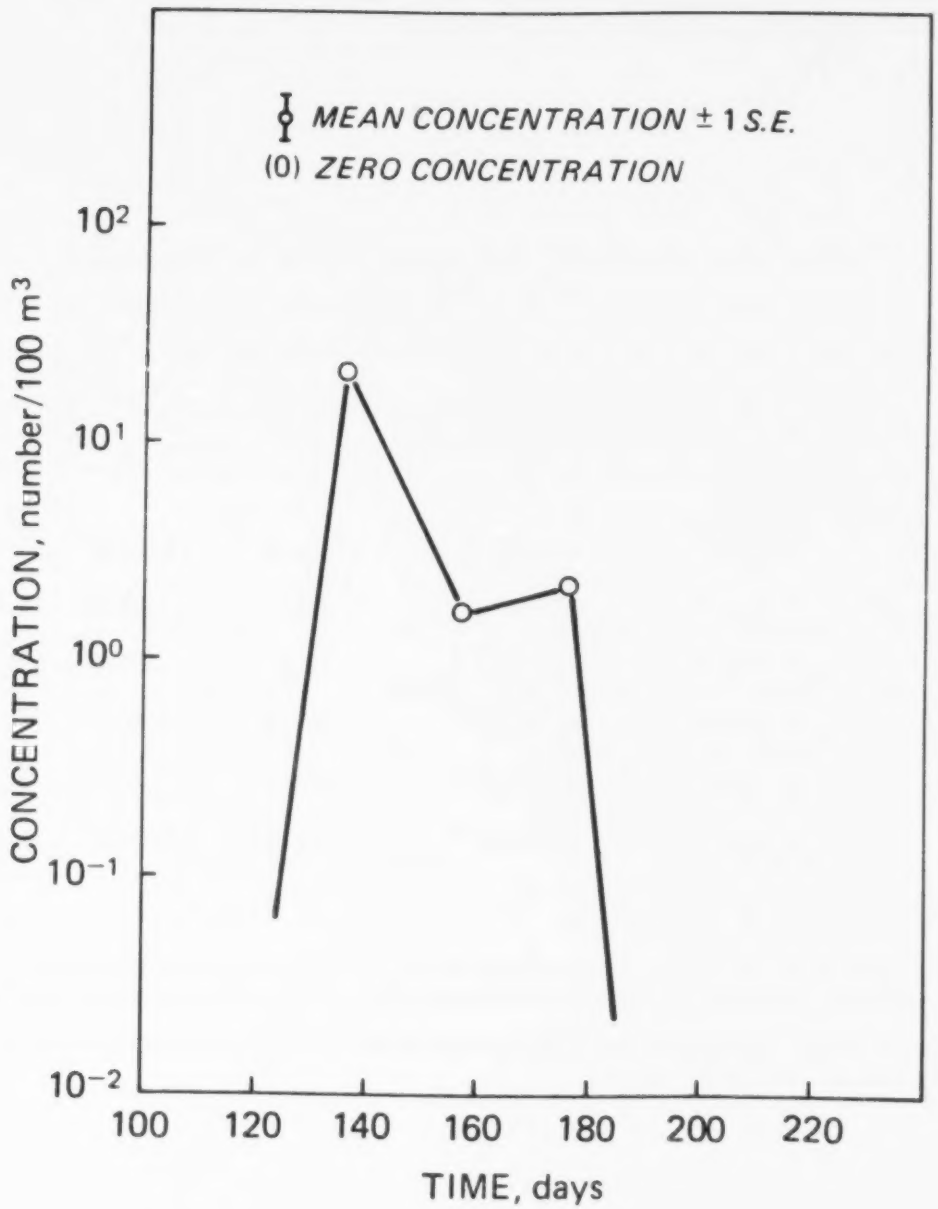

Figure 17. Larval perch concentration in 0 to 2 meter zone, Maumee Bay (1975).
Data Source: Ref.(5).

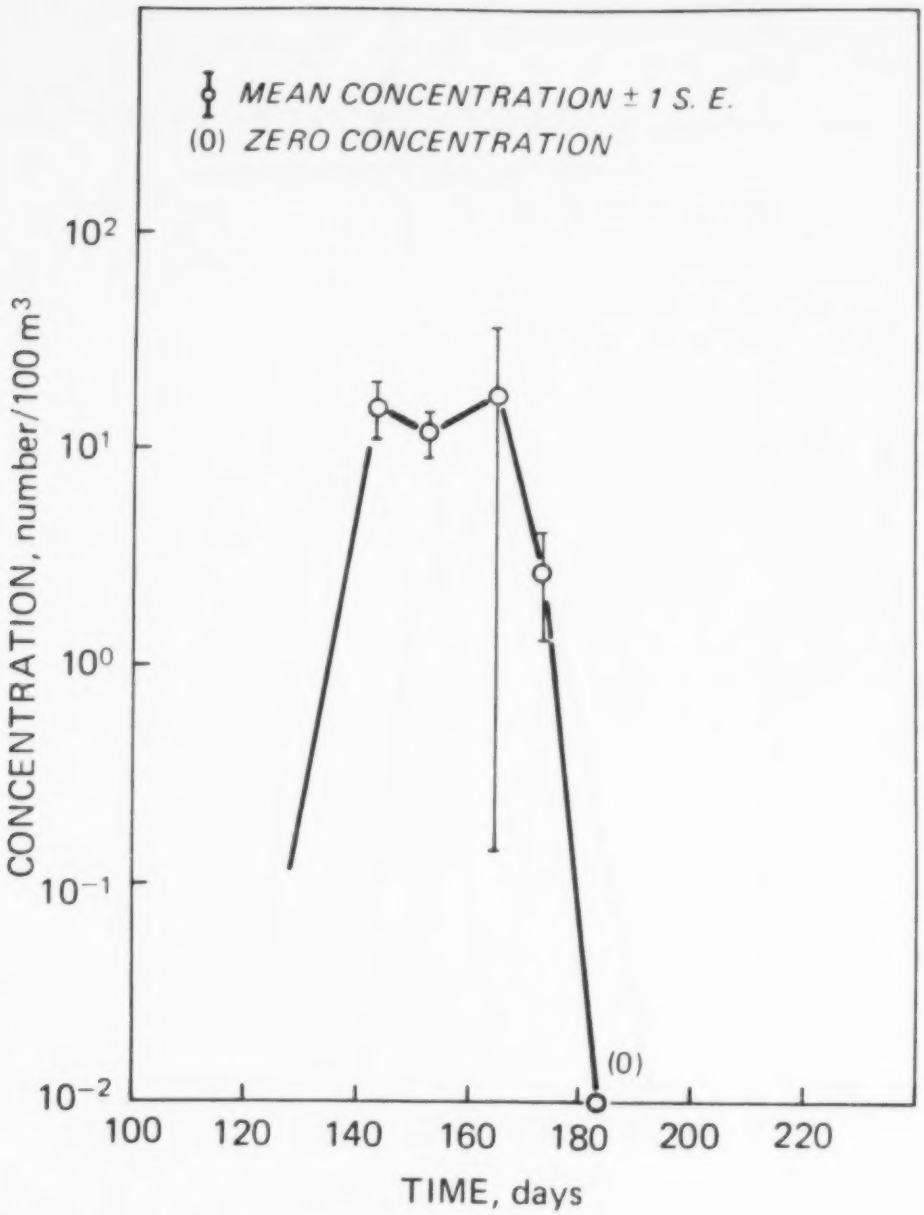

Figure 18. Larval perch concentration in 0 to 2 meter zone, Ohio Area A (1975).
Data Source: Table 12A.

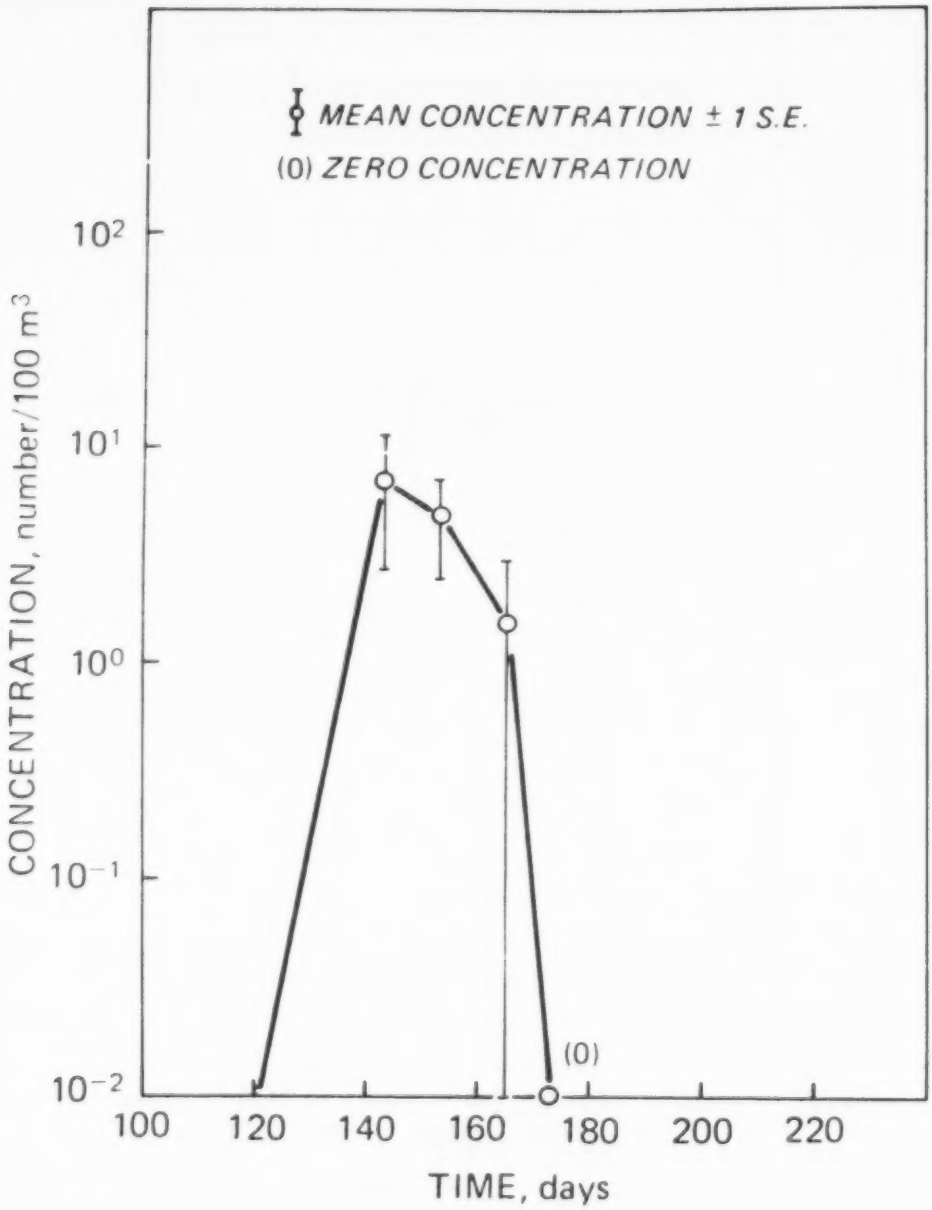

Figure 19. Larval perch concentration in 2 to 4 meter zone, Ohio Area A (1975).
Data Source: Table 12A.

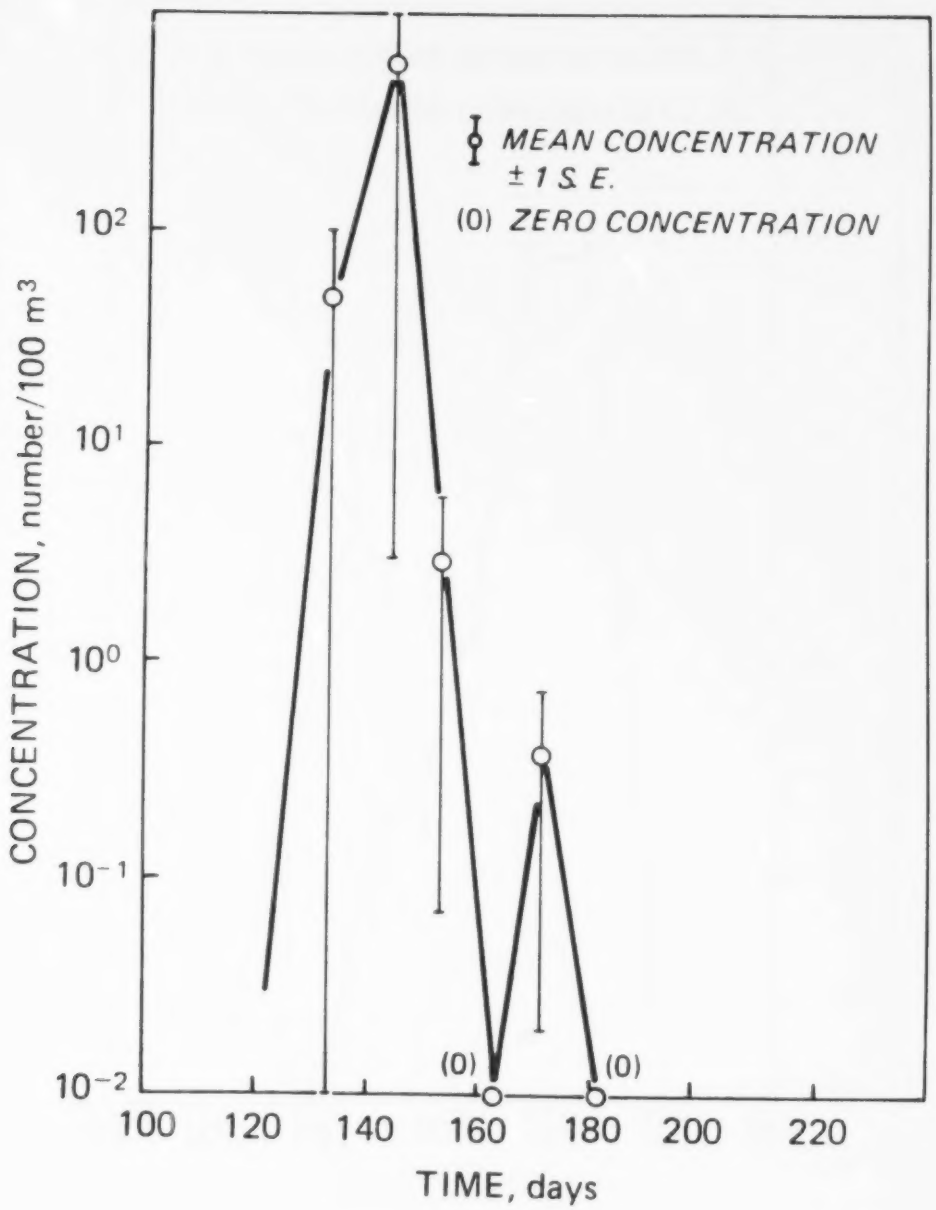

Figure 20. Larval perch concentration in 0 to 2 meter zone, Ohio Area C (1975).
Data Source: Table 12A.

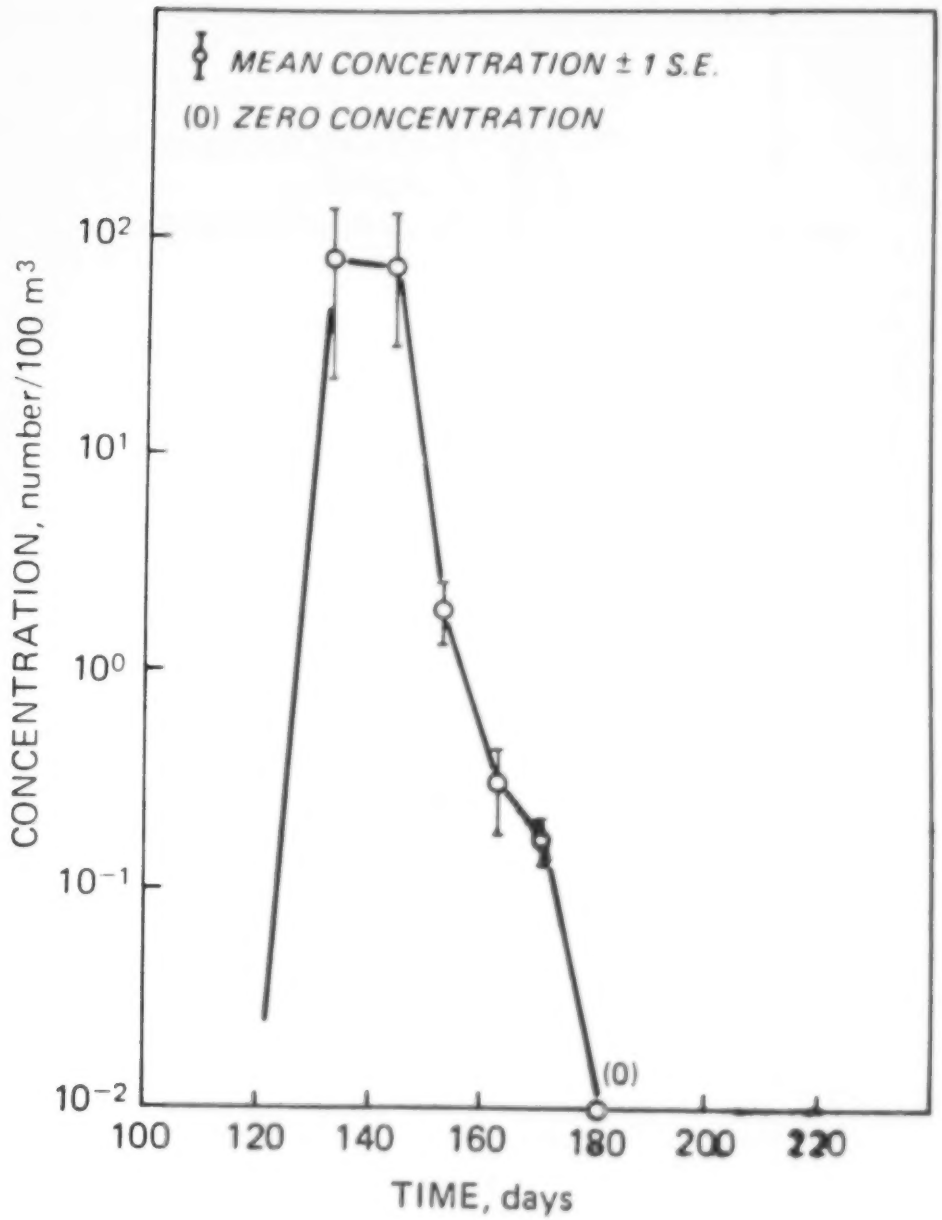

Figure 21. Larval perch concentration in 2 to 4 meter zone, Ohio Area C (1975).
Data Source: Table 12A.

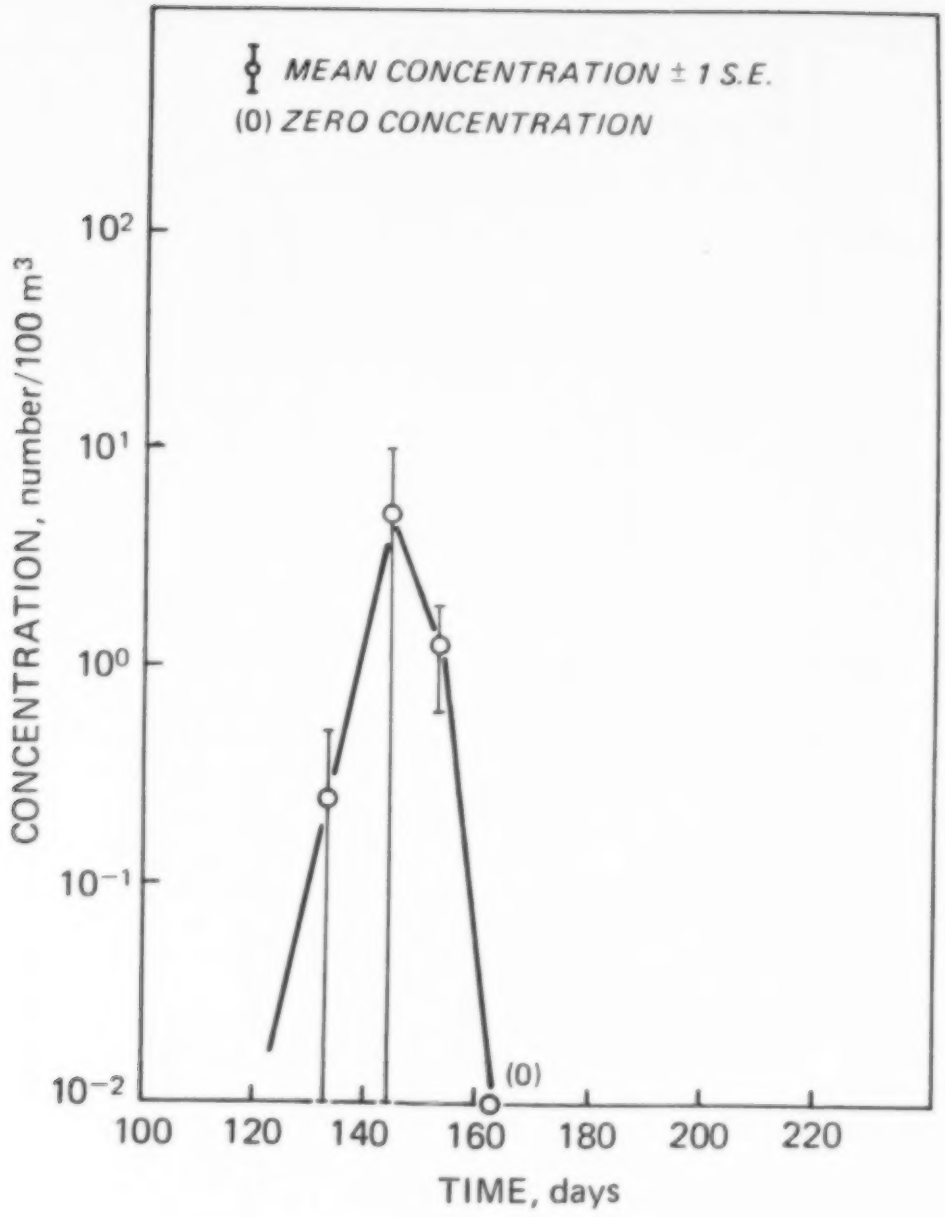

Figure 22. Larval perch concentration in 0 to 2 meter zone, Ohio Area D (1975).
Data Source: Table 12A.

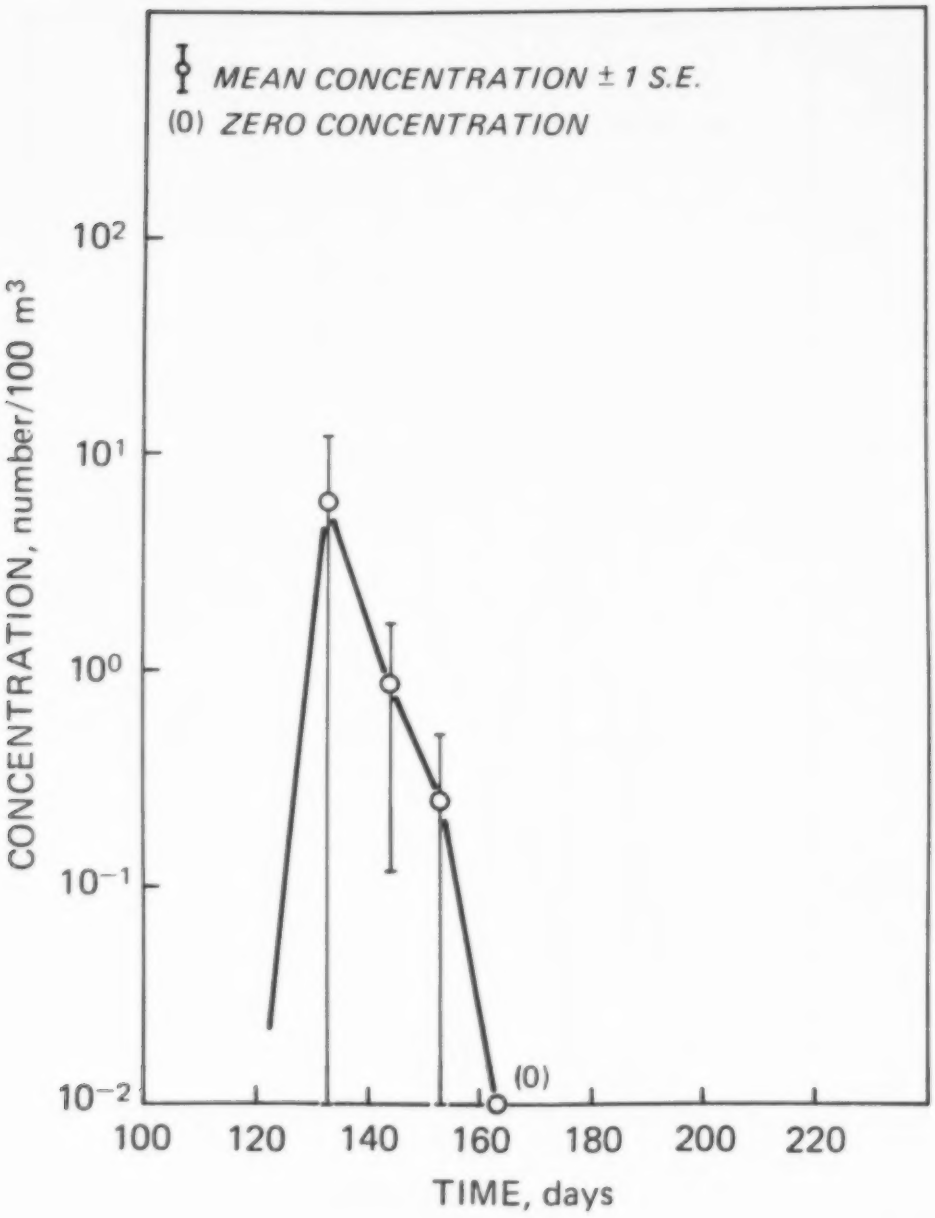

Figure 23. Larval perch concentrations in 2 to 4 meter zone, Ohio Area D (1975).
Date Source: Table 12A.

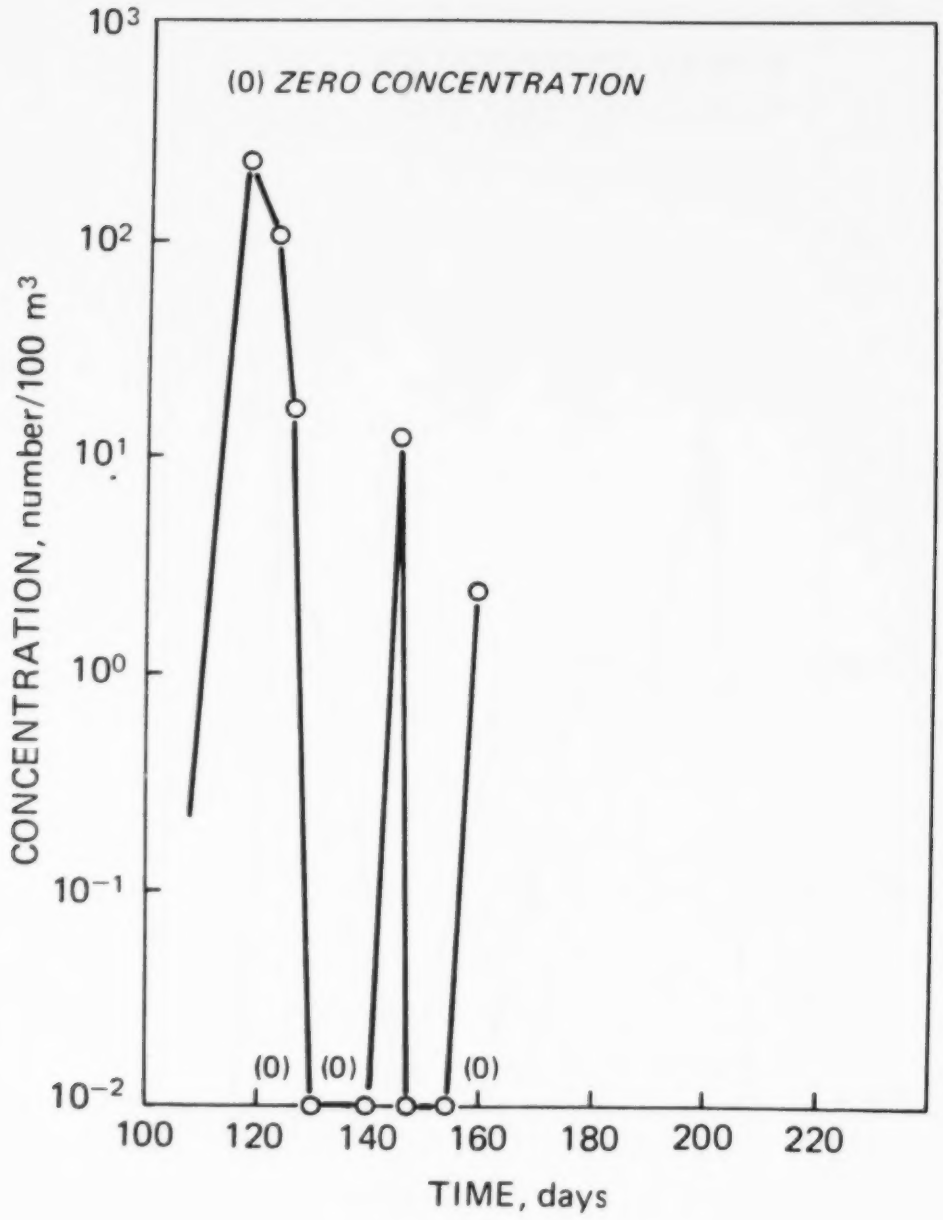

Figure 24. Larval perch concentration in 0 to 2 meter zone, Maumee Bay (1976).
Data Source: Ref.(6).

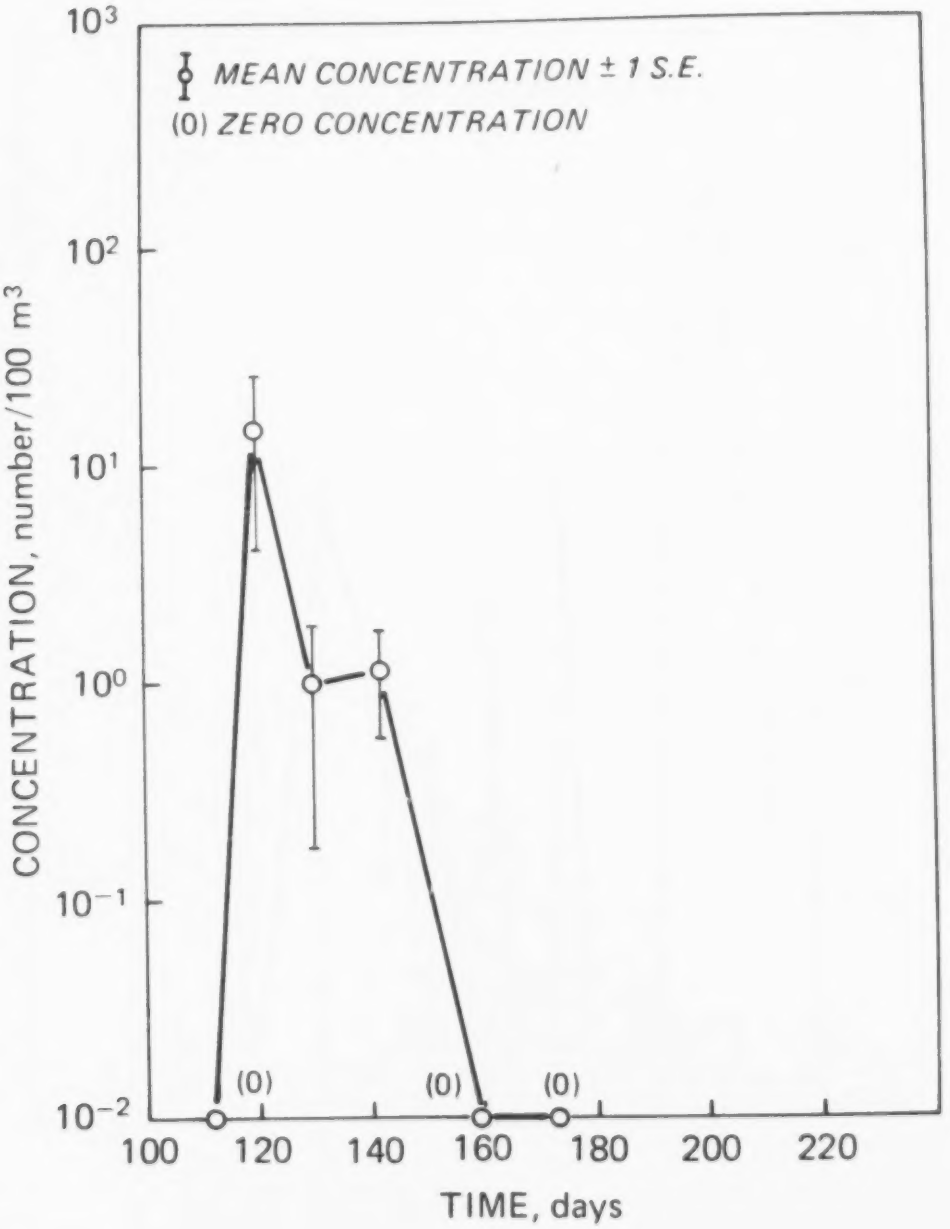

Figure 25. Larval perch concentration in 0 to 2 meter zone, Ohio Area A (1976).
Data Source: Table 12D.

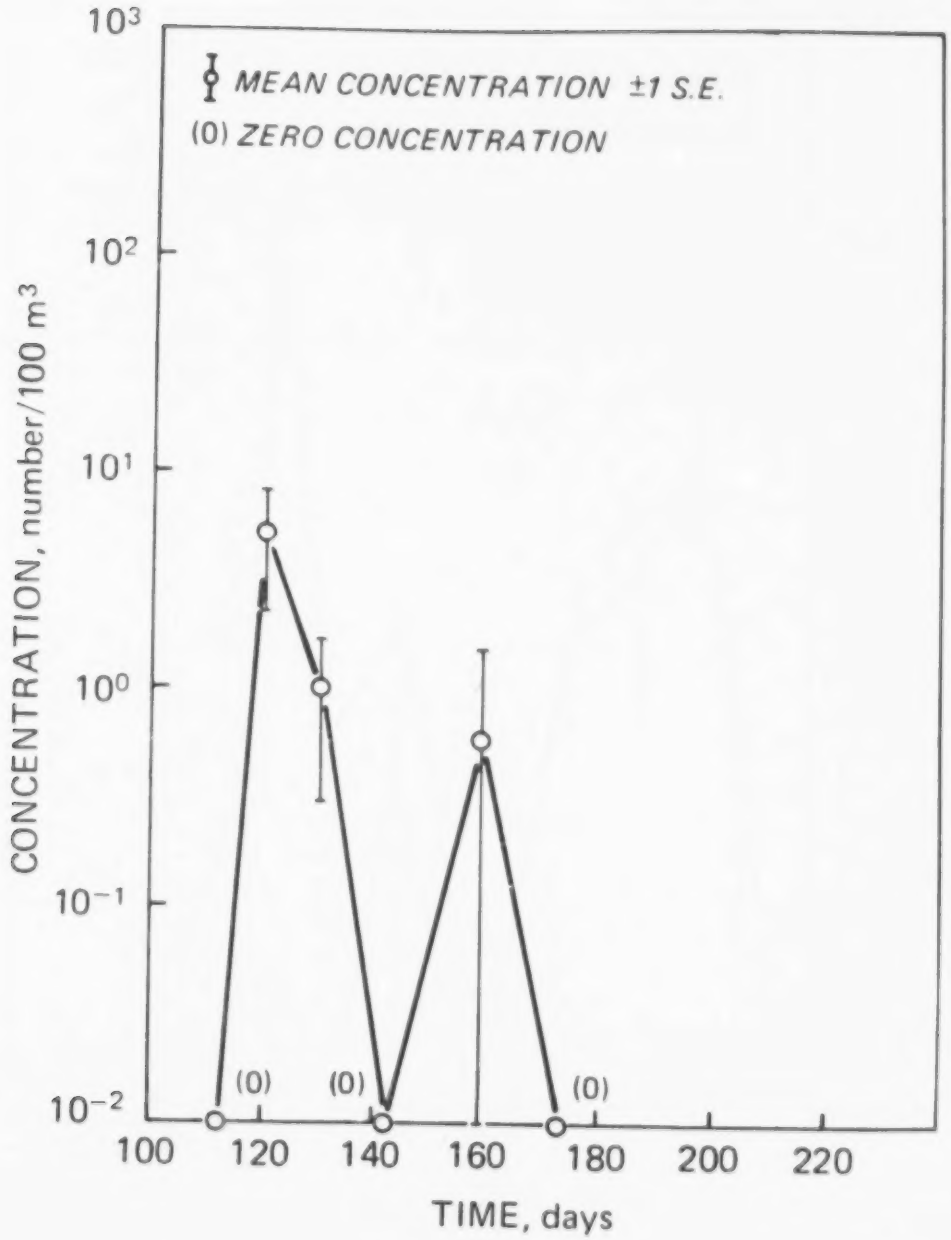

Figure 26. Larval perch concentration in 2 to 4 meter zone, Ohio Area A (1976).
Data Source: Table 12D.

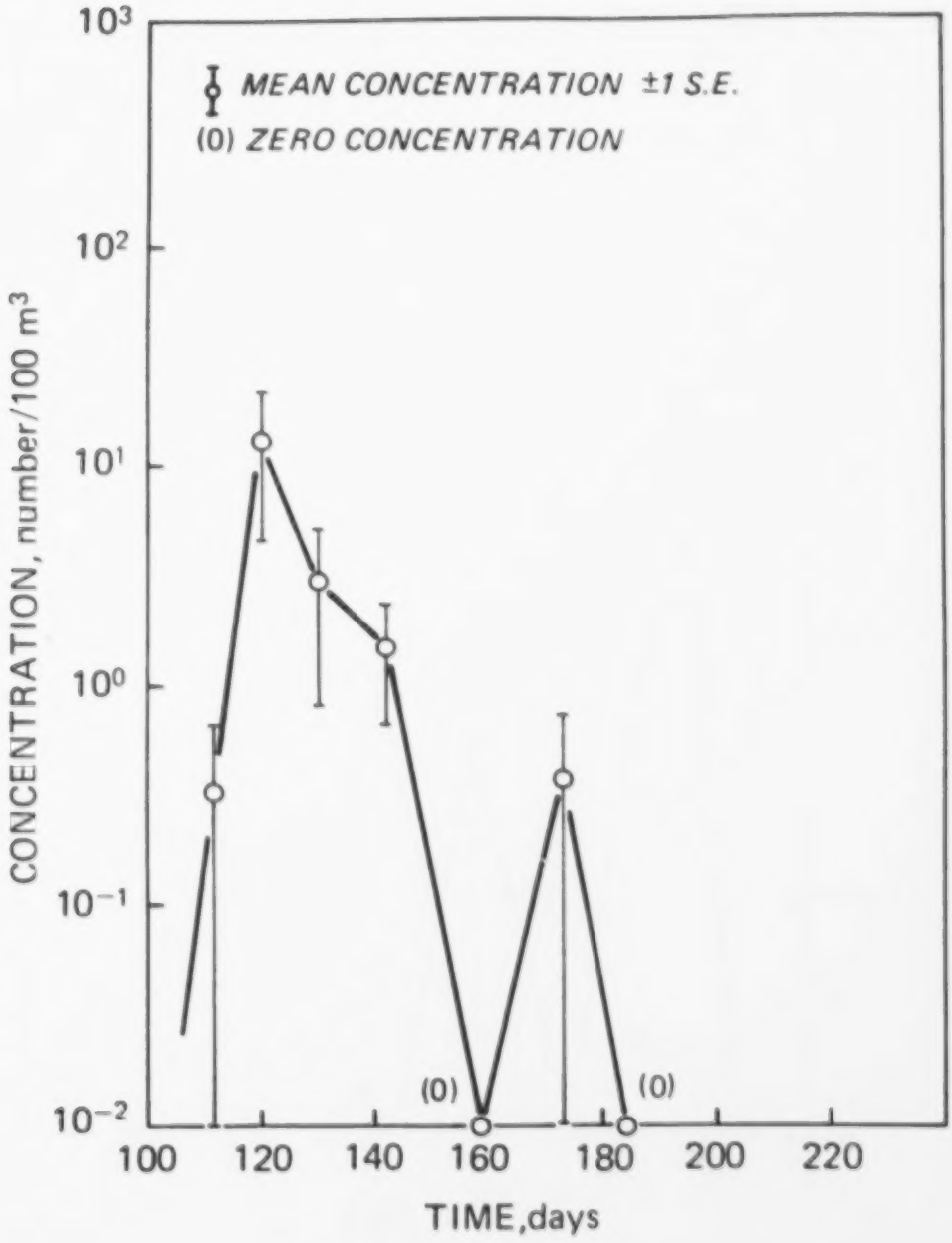

Figure 27. Larval perch concentration in 0 to 2 meter zone, Ohio Area C (1976).
Data Source: Table 12D.

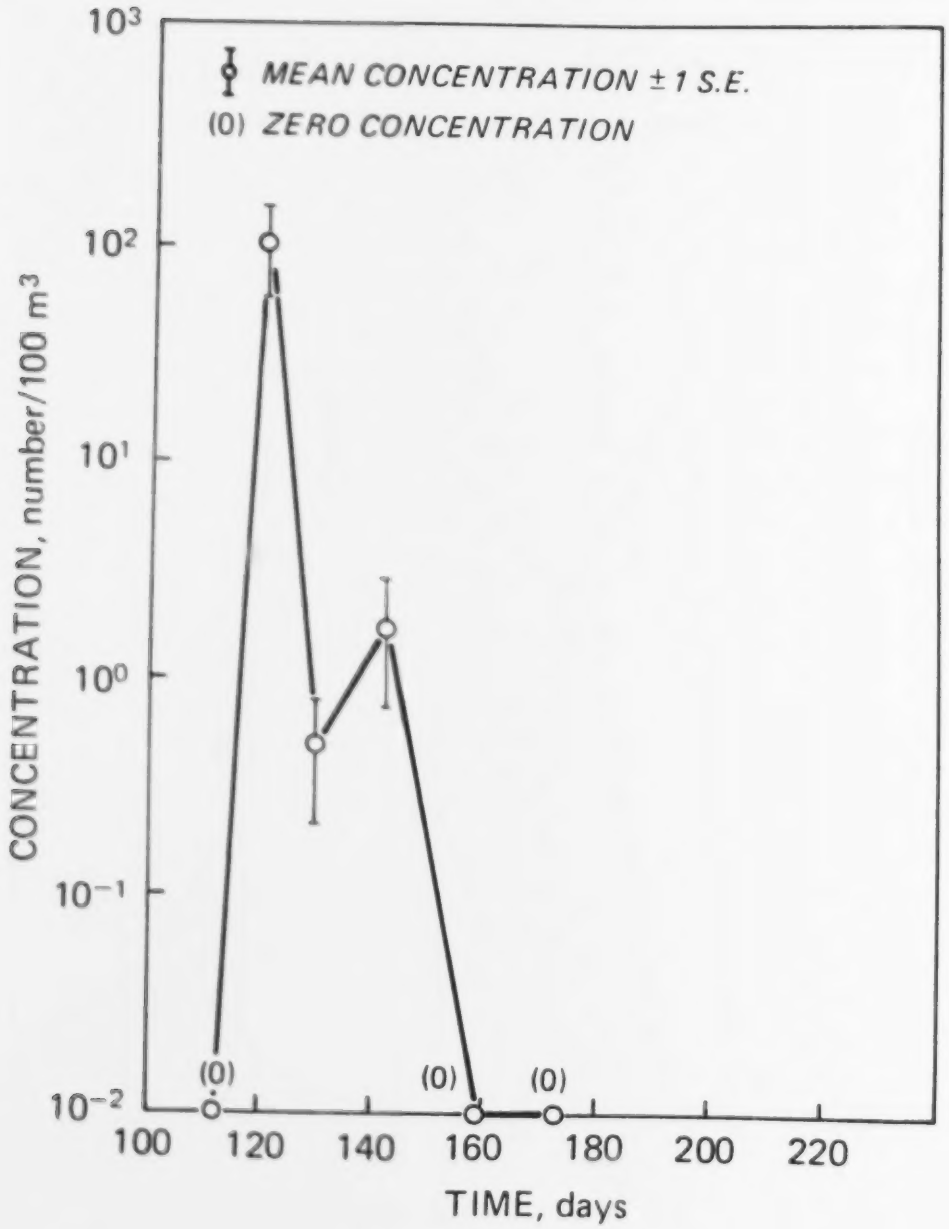

Figure 28. Larval perch concentration in 2 to 4 meter zone, Ohio Area C (1976).
Data Source: Table 12D.

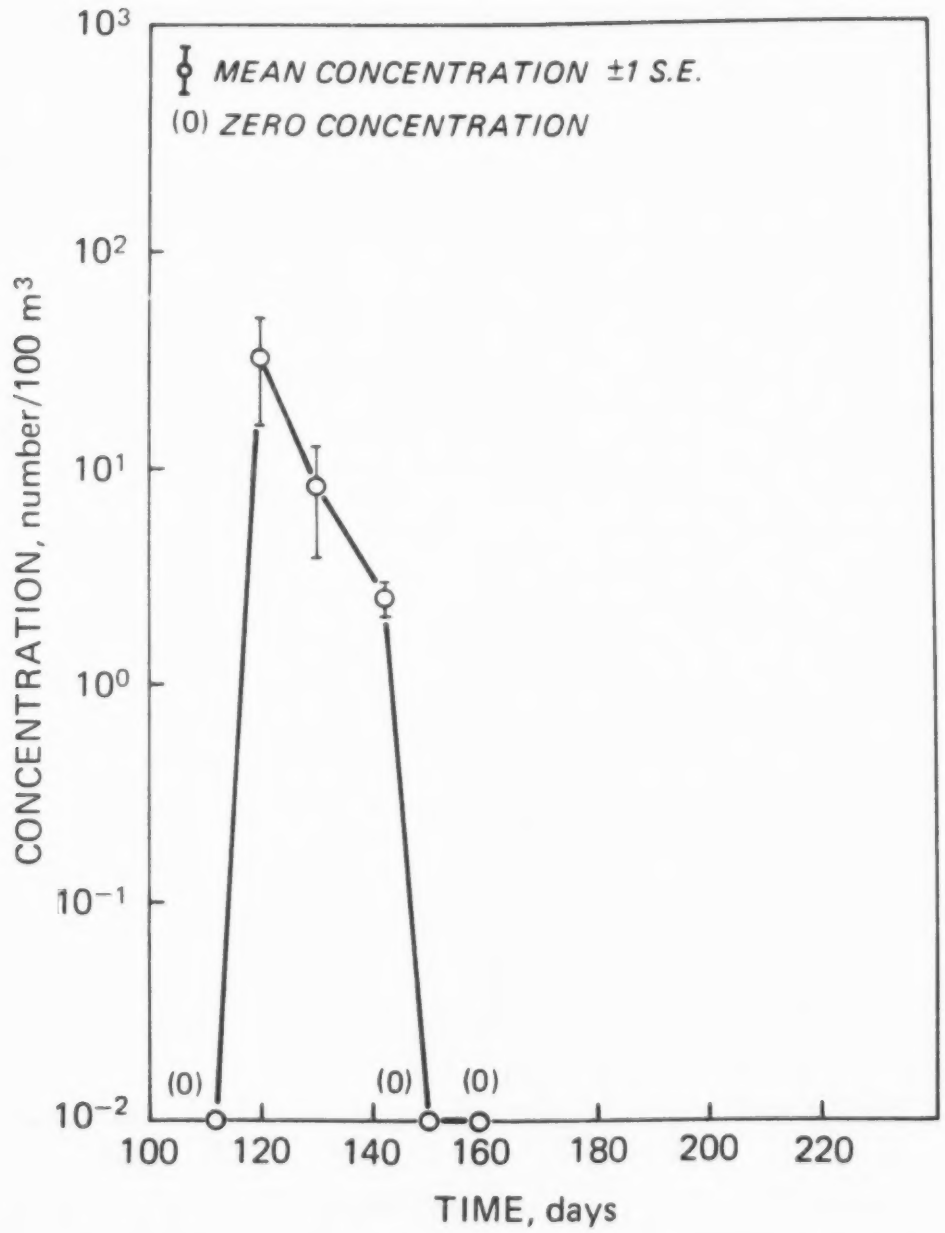

Figure 29. Larval perch concentration in 0 to 2 meter zone, Ohio Area D (1976).
Data Source: Table 12D.

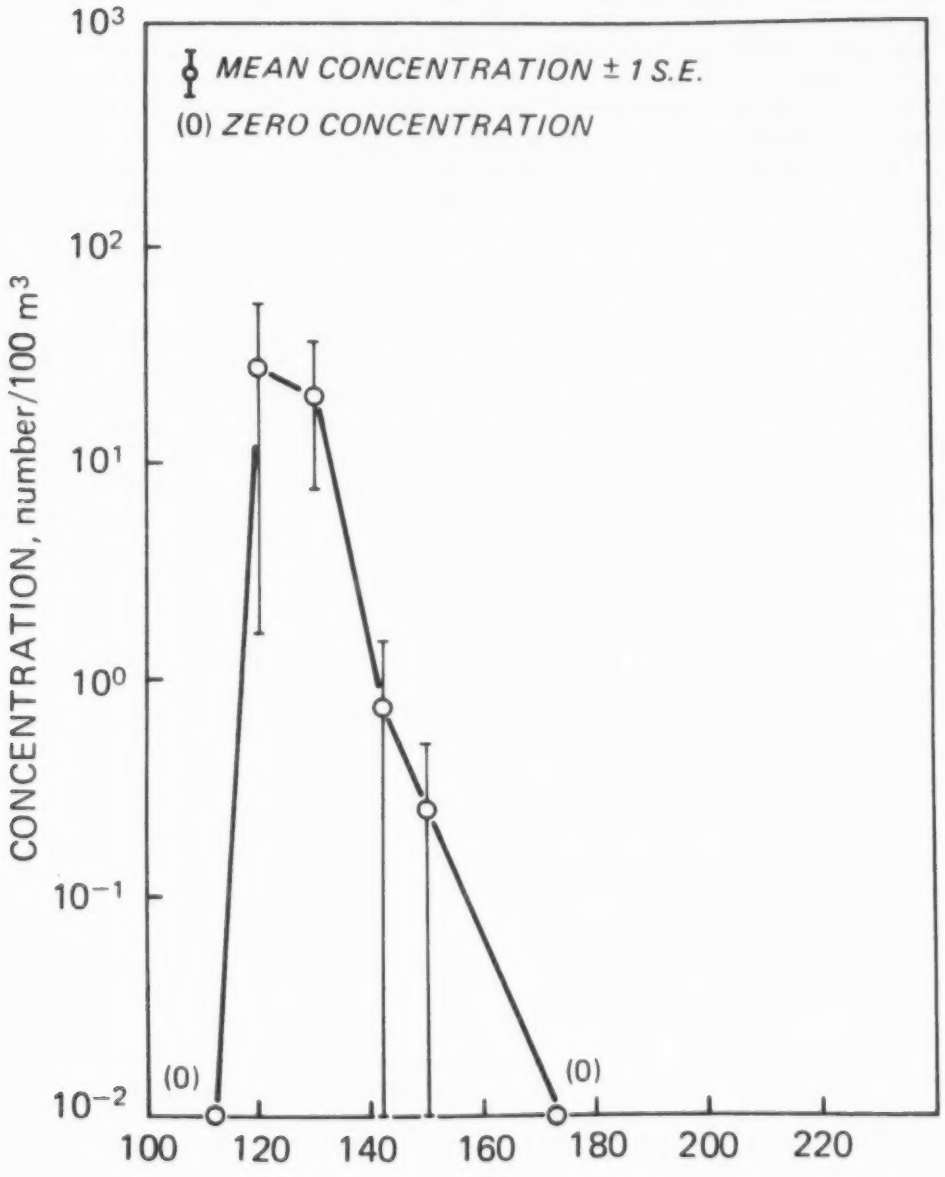

Figure 30. Larval perch concentration in 2 to 4 meter zone, Ohio Area D (1976).
Data Source: Table 12D.

Factors Affecting Larval Abundance

It is not within the scope of this analysis to elucidate the relative influences of biotic and environmental factors which determine larval production and subsequent strength of the year class. For present purposes, it is sufficient to summarize all such influences in terms of production (as defined above), natural mortality, recruitment into the young-of-year stage, entrainment by water intakes, and migration due to transport by the water column. Larval production occurs from hatching of eggs spawned directly in basin waters or in tributaries, estuaries, and shallow embayments which feed into basin waters. Perch larvae are transported initially from spawning beds into deeper waters by water motions and later by their own locomotion as well. Lateral movement is passive for the first days of life, although larvae exhibit very early a pattern of diurnal vertical migration which is undoubtedly not entirely passive (4). After a two to four week period of relatively intense production, yellow perch spawning activity rapidly drops, but it can occur even in mid-summer. Since Michigan-Ohio waters of the western basin form an open system, water and biota are exchanged with the Canadian portion of the western basin and the central basin of Lake Erie. Water, biotic, and abiotic materials are fed into Michigan-Ohio waters from numerous streams and two large rivers. The Detroit River supplies ninety-five percent of the total stream flow and transports yellow perch larvae into the western basin. Withdrawals for municipal and industrial uses cause losses of both water and its suspended constituents, including fish larvae (11). No separate estimates were made of numbers of larvae entering the basin from streams or rivers. Such estimates are not strictly necessary because measurements of larval concentrations in basin waters at several points will include larvae from streams and rivers, provided they reach the sampling zones prior to recruitment into the juvenile stage.

SECTION 4

METHODS

Approaches to Modeling Larval Production and Abundance

Ichthyoplankton abundance can be quantified by simulating the spawning population or by using time variable mathematical functions fitted to abundance measurements. Functional forms involving polynomials, rational functions, or exponentials may be assumed in which one or more parameters in the function are estimated from the data. One such model is specified by an equation of the form:

$$A(t) = \int_0^t P(t-x) \cdot s(t,x)\, dx$$

where:

$A(t)$ = larval abundance at time t ($t \geq o$),

$P(y)$ = instantaneous larval production rate at time instant y ($y \geq o$)

$s(t,x)$ = fraction of larvae produced in time interval t-x, t-x + dx) that survive a time interval of length x.

A variation of the above is:

$$A(t) = V \cdot C \cdot \int_0^t k \cdot e^{-(t-x)} \cdot (1-e^{-\beta x})\, dx$$

where:

$A(t)$ = larval abundance at time t

V = volume of reference basin (number of 100 M^3 units)

C = Mean total number of larvae per 100 M^3 deposited in reference volume during period of production

α = mortality related parameter

k = normalizing constant.

β = production related parameter

Difficulties with this approach are: a) the parameters may not be interpretable in terms of biological or environmental processes, b) conservation of larval numbers is not necessarily guaranteed. The approach followed below is based upon a materials balance for the net daily rate of change of larvae in a reference volume. Each source or sink for addition or removal of larvae is represented by an individual term; after both sides of the equation are divided by the size of the reference volume, a differential equation expressing the net rate of change in concentration is obtained. The equation contains two parameters, representing production and natural mortality of larvae. Concentrations in Michigan and Ohio waters are analyzed separately; therefore, two distinct reference volumes are used.

A Material Balance Model of Larval Abundance

A material balance formulation for the net daily rate of change in larval abundance for a specified reference volume is:

$$\dot{N}(t) = h(t) - v(t) - r(t) - m(t) - L(t) - E(t) \qquad (3)$$

where:

$\dot{N}(t)$ = net daily instantaneous rate of change in larval abundance in specified reference volume on day t. $(0 \leq t \leq 365)$.

$N(t)$ = number of larvae in reference volume on day t. $(0 \leq t \leq 365)$.

$E(t)$ = daily rate of loss of larvae from reference volume due to entrainment by condensor cooling waters of Edison power plant at Monroe, Michigan.

$L(t)$ = daily rate of loss of larvae in reference volume due to withdrawal of water by other industrial and municipal water intakes.

$h(t)$ = daily rate of addition of larvae to reference volume (daily production rate).

$r(t)$ = daily rate of recruitment of larvae in reference volume into the first juvenile stage of development (assumed to occur after 25th day of life following hatching).

$m(t)$ = daily rate of loss of larvae in reference volume due to natural mortality.

$v(t)$ = daily net emigration of larvae across boundary of reference volume due to water transport or larval locomotion.

Losses Due to Natural Mortality

Environmental conditions, natural predation and biotic factors which cause mortality among yellow perch larvae within the reference volumes

(Michigan waters and Ohio waters of western basin) are represented by a natural mortality parameter p:

p = mean daily fractional mortality rate for yellow perch larvae within the specified reference volume.

Natural mortality is assumed to be a force operative on all larvae alike where the chance of a given larva surviving a short interval Δt of time is $p \cdot \Delta t$, i.e., proportional to the length Δt. This assumption leads to a first order decay of the surviving population and the exponential survival function e^{-pt}. Equivalently, natural mortality is assumed to be proportional to abundance:

$$m(t) = p \cdot N(t) \qquad (4)$$

from which one deduces

$$N(t) = -m(t) = -p \cdot N(t),$$

$$N(t) = N(0)e^{-pt}.$$

Thus, the proportion of larvae surviving t days following hatching on day 0 is:

$$\frac{N(t)}{N(0)} = e^{-pt}$$

The mortality parameter, p, is estimated by fitting a solution to equation (3) to separate field based estimates of mean concentration of larvae in Michigan and Ohio waters. The assumption that p is a constant is interpreted to mean that total conditions that produce larval mortality in a given year remain unchanged. On the average, throughout the months May to August, the fraction of remaining larvae that do not survive from one day to the next fluctuates about a constant p. This is equivalent to the assumption that the natural mortality rate on larval production within a given spawning season is conditionally independent, but it implies nothing about a possible variation in p from one year to the next, which may reflect changes in larval production or other biotic or environmental factors.

Production of Yellow Perch Larvae

Larval production occurs from the hatching of eggs spawned directly in Michigan-Ohio waters of the basin and by larvae transported into the basin from tributaries, estuaries and across the international boundary from Canadian waters.[1] Approximately six to twelve days following spawning, eggs hatch and an individual yolk-sac or pro-larvae begins the first day of

[1]Limits on annual production are estimated in terms of numbers of female spawners, number of eggs deposited, population size of species, and hatching success. An upper limit is estimated to be approximately 7 to 8 billion.

its life. It is evident from examination of field samples (Figure 14) that production rapidly builds up to a peak, remains elevated for a period of time, decreases to a low level for an additional period, then completely ceases. Any mathematical function h(t) used to describe larval production should distribute the pro-larval input over approximately the same period in which pro-larvae are observed in the reference volume. The function h(t) should peak at approximately the same time that peak production is estimated to occur in the reference volume, and it should exhibit rate of change characteristics suggested by field data (Figures 13 to 16). Finally, it should contain a parameter describing productive intensity which can be estimated from field data. A function which meets the above criteria is:

$$h(t) = \begin{cases} 0 & (0 \leq t \leq T_o) \\ B \cdot h \cdot \binom{m}{0} \cdot q^0 \, (1-q)^{m-0} & (T_o < t \leq T_o + d) \\ B \cdot h \cdot \binom{m}{1} \cdot q^1 \, (1-q)^{m-1} & (T_o + d < t \leq T_o + 2d) \\ B \cdot h \cdot \binom{m}{2} \cdot q^2 \, (1-q)^{m-2} & (T_o + 2d < t \leq T_o + 3d) \\ \vdots & \vdots \\ B \cdot h \cdot \binom{m}{m-1} \cdot q^{m-1} \cdot (1-q)^1 & (T_o + (m-1)d < t \leq T_o + md) \\ B \cdot h \cdot \binom{m}{m} \cdot q^m \cdot (1-q)^0 & (T_o + md < t \leq T_o + T_1) \\ 0 & (T_o + T_1 < t \leq 365) \end{cases} \quad (5)$$

where:

$m + 1$ = maximum number of time periods in which production can occur.

d = number of days in each of the $m + 1$ time periods of larval production.

q = parameter which determines the time period in which production function peaks. $(0 < q < 1)$. q and m jointly determine the spread or skewness of the production function over the period of larval abundance.

h = production parameter or mean total number of larvae deposited per 100 cubic meters of water in reference volume. The parameter h directly influences the amplitude of the production rate.

T_o = day on which production begins.

T_1 = maximum number of days that production occurs.

B = number of 100 M^3 unit volumes of water in reference volume.
(= 4.976×10^6 for Michigan waters)
(= 9.393×10^7 for Ohio waters when Maumee estuary is included; 9.351×10^7 if zones A,B,C,D, and E alone are considered).

$\binom{m}{x}$ = binomial coefficient.

Total larval production for d consecutive days in period x in the reference volume is therefore:

$$d \cdot B \cdot h \cdot \binom{m}{x-1} q^{x-1} \cdot (1-q)^{m-x+1}$$

$$(x = 1,2, \cdots, m + 1)$$

Total larval production is distributed over the periods 1,2, $\cdots$, m+1) in the reference volume and sums to:

$$\sum_{x=1}^{m+1} d \cdot B \cdot h \cdot \binom{m}{x-1} q^{x-1} \cdot (1-q)^{m-x+1} =$$

$$d \cdot B \cdot h \sum_{x=1}^{m+1} \binom{m}{x-1} q^{x-1} \cdot (1-q)^{m-x+1}$$

From probability theory:

$$\sum_{x=1}^{m+1} \binom{m}{x-1} q^{x-1} (1-q)^{m-x+1} = 1,$$

so that total production in the reference volume during the period of abundance for any given year is:

$$\text{Total Production} = d \cdot B \cdot h \qquad (6)$$

The function h(t) has the shape of a series of stair steps which can be "upstairs", "downstairs" or "up and down stairs," depending upon the values of m and q. The height of each step is proportional to the value of h. Since h(t) as defined by equation (5) contains m+1 discontinuous steps the particular solution to equation (3) which incorporates a production function defined by equation (5) must reflect these discontinuities by being solved explicitly and separately for each of the m+1 sub-intervals of time during which production can occur.

The parameters q and m are determined together on a trial and error basis (visual inspection aided by computer calculations) by selecting values which cause h(t) to exhibit a similar production gradient and to peak at approximately the same time that larval abundance is estimated to reach a maximum. The values selected for q and m can indirectly affect the value of the production parameter, h, obtained by fitting (by least squares) the solution to equation (3), containing h and p, to the estimated concentra-

tions in the reference volumes shown in Figures 7, 13, 15, and 16. As q and m are estimated, values of d are determined by inspection of Figures 7, 13, 15, and 16 (one value of d for each case) so that the quantity $(m+1)\cdot d$ matches the length of the period over which larval production is estimated to have occurred.

Production in Ohio Water: 1975

From inspection of field survey data and Figure 15, production is estimated to have commenced between May 1 and May 10, continued at a high rate until approximately May 21 (day 144), and then declined rapidly. Larval perch are fully recruited (by assumption) into the young-of-year stage 25 days after their day of production. Larval abundance peaks on approximately day 144, so that nearly all production must have occurred on or before that date. Tables of the binomial probability function show that when $m = 5$ and $q = 0.10$, and setting $d = 7$, 59 percent of production occurs in the first 7 days, and 33 percent occurs from the 7th to 14th day, or a total of 92 percent of production by the 14th day. If production commences on day 127, the 17th day of production occurs on day 144, the day of approximate peak larval abundance, and the 35th and final day of production occurs on day 162, 25 days prior to the day on which all larvae are assumed to have been fully recruited into the young-of-year stage (after inspection of field sampling records). Therefore, by selecting the binomial probability function corresponding to $m = 5$ and $q = 0.10$, the following production function is obtained as a special case of equation (5):

$$h(t) = \begin{cases} 0 & 0 \leq t \leq 127 \ (=T_o) \\ 0.5905 \cdot B \cdot h & 127 < t \leq 134 \ (=T_o + d) \\ 0.3280 \cdot B \cdot h & 134 < t \leq 141 \ (=T_o + 2d) \\ 0.0729 \cdot B \cdot h & 141 < t \leq 148 \\ 0.0081 \cdot B \cdot h & 148 < t \leq 155 \\ 0.0004 \cdot B \cdot h & 155 < t \leq 162 \\ 0 & 162 < t \leq 365 \end{cases} \qquad (7)$$

where:

$$B = 9.393 \times 10^7$$

$$d = 7$$

Other combinations of m and q were tested but none yielded a distribution which so adequately fit the field observations on the spread and apparent timing of peak production. That is, equation (7) together with alternatives generated by varying p, d, and m, were compared by substitution into

equation (3). Equation (7) produced a much superior fit when the resulting solution to equation (3) was matched to the data shown in Figure 15. (See Figure 36 for optimum values of p for selected values of h). It is clear, therefore, that numerical analysis of two or more candidate production functions may be necessary in order to select the function which most adequately describes the actual but unknown time dependent introduction of larvae into the reference volume. The end result is a more reliable estimate of total production and the conditional relationship of natural mortality to total larval production.

Analyses following the same lines as the preceding case led to production functions describing larval perch production for the three remaining cases:

Production in Ohio Waters: 1976

$$h(t) = \begin{cases} 0 & 0 \leq t \leq 106 \\ 0.5905 \cdot B \cdot h & 106 < t \leq 113 \\ 0.3280 \cdot B \cdot h & 113 < t \leq 120 \\ 0.0729 \cdot B \cdot h & 120 < t \leq 127 \\ 0.0081 \cdot B \cdot h & 127 < t \leq 134 \\ 0.0004 \cdot B \cdot h & 134 < t \leq 141 \\ 0 & 141 < t \leq 365 \end{cases} \qquad (8)$$

$B = 9.393 \times 10^7$

$d = 7$

Production in Michigan Waters: 1975

$$h(t) = \begin{cases} 0 & 0 \leq t \leq 120 \\ 0.4437 \cdot B \cdot h & 120 < t \leq 134 \\ 0.3915 \cdot B \cdot h & 134 < t \leq 148 \\ 0.1382 \cdot B \cdot h & 148 < t \leq 162 \\ 0.0244 \cdot B \cdot h & 162 < t \leq 176 \\ 0.0001 \cdot B \cdot h & 176 < t \leq 190 \\ 0 & 190 < t \leq 365 \end{cases} \qquad (9)$$

$$B = 4.976 \times 10^6$$

$$d = 14$$

Production in Michigan Waters: 1976

$$h(t) = \begin{cases} 0 & 0 \leq t \leq 106 \\ 0.4437 \cdot B \cdot h & 106 < t \leq 120 \\ 0.3915 \cdot B \cdot h & 120 < t \leq 134 \\ 0.1382 \cdot B \cdot h & 134 < t \leq 148 \\ 0.0244 \cdot B \cdot h & 148 < t \leq 162 \\ 0.0022 \cdot B \cdot h & 162 < t \leq 176 \\ 0.0001 \cdot B \cdot h & 176 < t \leq 190 \\ 0 & 190 < t \leq 365 \end{cases} \quad (10)$$

$$B = 4.976 \times 10^6$$

$$d = 14$$

Recruitment into the Young-of-Year Stage

Following a period of maturation lasting from 20 to 30 days, the surviving larvae are recruited into the first juvenile stage of development or young-of-year stage. (The length of the larval stage is defined as D = 25 days for all calculations below). Upon consideration of the effect of natural mortality upon the number of young-of-year recruits it can be deduced that the recruitment rate, r(t), is approximately equal to a time translation of the production rate, h(t), reduced in amplitude by the factor e^{-25p} which accounts for natural mortality that occurs during the 25 day period of maturation. Therefore:

$$r(t) = h(t-25)e^{-25p} \quad (11)$$

If the fact that larvae killed by entrainment in water intakes or other point sources will not be recruited into the young-of-year stage is not taken into account in equation (11), the estimate of recruitment provided by equation (11) may be slightly exaggerated. Although equation (11) accounts for removal of larvae that survive 25 days from the total pool existing at time t, the actual effect represented by equation (11) and observed in the field is a reduction in the number of 20 to 30 day old larvae due to their enhanced ability to avoid capture by sampling gear. Because the ability of larvae to avoid capture by sampling gear does not jump from zero to 100 percent effectiveness at the exact age of 25 days, equation

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(11) is only an approximation of the process of net avoidance. Since a more accurate specification of an avoidance function cannot be verified, further refinement of equation (11), taking into account ability of larvae to avoid capture as a function of their size, is not attempted.

Emigration

The term v(t) accounts for emigration of larvae across the international boundary and between Michigan and Ohio waters. The patterns of circulation of the water mass in the western basin are known and studies of larval transport using a hydrodynamic model of Lake Erie (10) suggest a net export of larvae out of Michigan territorial waters. Numerical studies show that larvae produced along the Michigan shoreline can be removed from Michigan waters in as few as two days. Larvae produced in Maumee Bay are transported into both Michigan and Ohio waters, but under normal southwest wind conditions during late spring, most are exported into Michigan waters. Larvae which enter the western basin from the Detroit River are transported into Michigan waters as well. Thus, both in-and-out migration of larvae occurs in Michigan waters. (In-migration is accounted for in the production term h(t).) Numerical studies suggest that by the tenth day after larvae are hatched within one kilometer of the Michigan shoreline, up to fifty percent could be transported into international waters unless they are killed or their own lateral swimming motion counteracts water circulation. It is estimated that for Michigan waters net out migration of perch larvae occurs but may not be more than 5 to 10 percent of total production. Large numbers of larvae are lost through natural mortality before they reach the 8 to 10 meter depth zone near international waters after having been hatched along the Michigan shoreline. A total net loss due to emigration reduces abundance on any given day and consequently affects estimates of the parameter h. Larval concentrations sampled in Ohio waters (5), and in zone F (Canadian waters) combined with numerical simulation studies of water circulation in the western basin (10) indicate a net loss of larvae from Ohio waters due to advective transport. Soon after hatching, perch larvae migrate vertically in the water column (4), and as a result they become vulnerable to transport by near-surface currents which carry them into the midwaters of the basin. As they settle to the bottom, however, their direction of transport is reversed and they move further into Michigan-Ohio waters. Numerical studies indicate considerable mixing of larvae from separate spawning areas. It is difficult to establish with confidence a percentage of larvae that are transported out of Ohio waters due to movement by the water column, but simulation studies (10) suggest that it is less than five percent.

Lateral migration of larvae by self-locomotion occurs but the extent to which it influences migration from or to Ohio-Michigan waters is unknown.

In the numerical analyses conducted in this study, emigration is assumed to be zero:

$$v(t) = 0 \qquad (0 \leq t \leq 365) \tag{12}$$

The effect is that any net loss in abundance due to emigration is confounded with production and natural mortality. That is, if emigration causes a reduction in abundance but is assumed to be zero (in the specification of the term v(t)) the estimated value of the production parameter h can be biased low. If an upper limit is placed upon emigration by assuming that:

$$v(t) < \alpha \cdot N(t)$$

where:

α = an assumed maximum mean daily fractional loss in abundance due to emigration

then, for any fixed estimate of the parameter h, the resulting optimum estimate of the parameter p is the sum of the mean daily natural mortality fraction and the mean daily emigration fraction. Given the mean daily emigration fraction the mean daily natural mortality fraction is determined by subtracting the value of the daily emigration fraction. The fraction α cannot be determined with precision but may be less than 0.005, i.e., one-half percent of daily abundance.

If emigration is treated as a function independent of abundance, then the assumption that $v(t) = 0$ when in fact $v(t) > 0$ leads to an underestimate of total production but may have no effect at all on the estimate of mean daily natural mortality fraction. The underestimation of emigration has exactly the same effect as the underestimation of mortality from water intake entrainment. It is believed that larval emigration losses are at most 5 to 10 percent of total production, so that if the production parameter h can be estimated assuming $v(t) = 0$, then emigration can be approximately accounted for by adding 10 percent to the value of h.

Larval Losses Due to Entrainment in the Monroe Power Plant Cooling Water Intake

Ichthyoplankton concentrations have been sampled at numerous locations in the immediate vicinity of the cooling water intake of the Detroit Edison power plant at Monroe (4); (also see Figure 8). The number of yellow perch larvae killed by entrainment is estimated by multiplying daily consumption of water by mean concentration of live larvae in the cooling water column, multiplying that product by the fraction of live larvae killed in the entrainment cycle, and summing the result over all days in which larvae are known to be present in the water column:

$$\begin{array}{l}\text{number larvae killed} \\ \text{in given year due to} \\ \text{cooling water} \\ \text{entrainment}\end{array} = \sum_{\substack{\text{days in period} \\ \text{of larval} \\ \text{abundance}}} \begin{array}{l}(\text{daily cooling water usage}) \times \\ (\text{concentration of live larvae}) \times \\ (\text{fraction of live larvae} \\ \text{killed})\end{array} \qquad (13)$$

Various estimates of total numbers killed in a given year can be obtained, depending upon how the terms on the right hand side of equation (13) are estimated. Appendix E illustrates four methods of estimating fraction of

live larvae killed due to the entrainment process. Daily cooling water usage is probably the most accurately known as records from which daily usage rates of cooling water (Figure 31) can be obtained are maintained at power plants.

Measurement of the concentration of larvae in the cooling water column is most subject to error and depends upon: a) location of the sampling station; b) frequency of sampling; c) time of day of sample; and, d) sampling gear. Figures 32 and 34 show concentrations sampled in 1974, 1975, and 1976 at the stations shown in Figure 8. Mean concentrations for the 0 to 6' depth zone in the Raisin River - Maumee Bay area for 1975 and 1976 are also plotted for comparison purposes. The lines shown in Figure 33 represent upper and lower values of larval concentrations. These values are used to estimate the number of larvae entrained during the period of abundance in 1975. It might be argued that Station 2 located in the upper discharge area represents the most uniformly mixed section of the water column and therefore should provide the most unbiased measurements on concentrations of larvae in the cooling water. However, substantial statistical fluctuations in larval concentrations occur at Station 2 as well as at all other stations (Table 16 and Figure 34); therefore, to ignore observations obtained at other stations is to make less than optimum use of the information contained in the full set of measurements. Based upon the upper and lower limits of concentration shown in Figure 33 and upon the published record of daily cooling water usage (Figure 31), lower and upper estimates of numbers entrained in 1975 were 2,726,000 and 14,262,000, respectively. Based upon an estimate that 20 percent of yellow perch larvae entering the cooling cycle are either dead or dying using data published by Cole (4, Table 9), the number of live larvae entrained is estimated to be between 2,180,800 and 11,409,600. Following methods 3 and 4 outlined in Appendix E and using larval mortality data published by Cole (op. cit.), estimates of the percentage of larvae killed due to the entrainment process are:

$$100p = 100\left(1 - \frac{1.4}{32} \cdot \frac{40}{5}\right) = 65 \qquad \text{(Method 3) (1975 data)}$$

and

$$100p = 100\left(1 - \frac{8}{3.6} \cdot \frac{5}{40}\right) = 72 \qquad \text{(Method 4) (1975 data)}$$

Therefore, using an estimate of 70 percent larval mortality due to entrainment, the lower and upper estimates of live larvae entrained and killed are 1,526,560 and 7,986,720, respectively. Inspection of perch larval concentrations in cooling water, which Detroit Edison published in 1975 (7) showed peak densities on day 156, approximately 25 days after the peak plotted in Figure 33, suggesting that larval perch concentrations in the cooling water column may have been substantially higher in the period 130 to 160 days than the values indicated by the solid lines in Figure 33. The mean daily rate of loss estimated to have occurred in 1975 is:

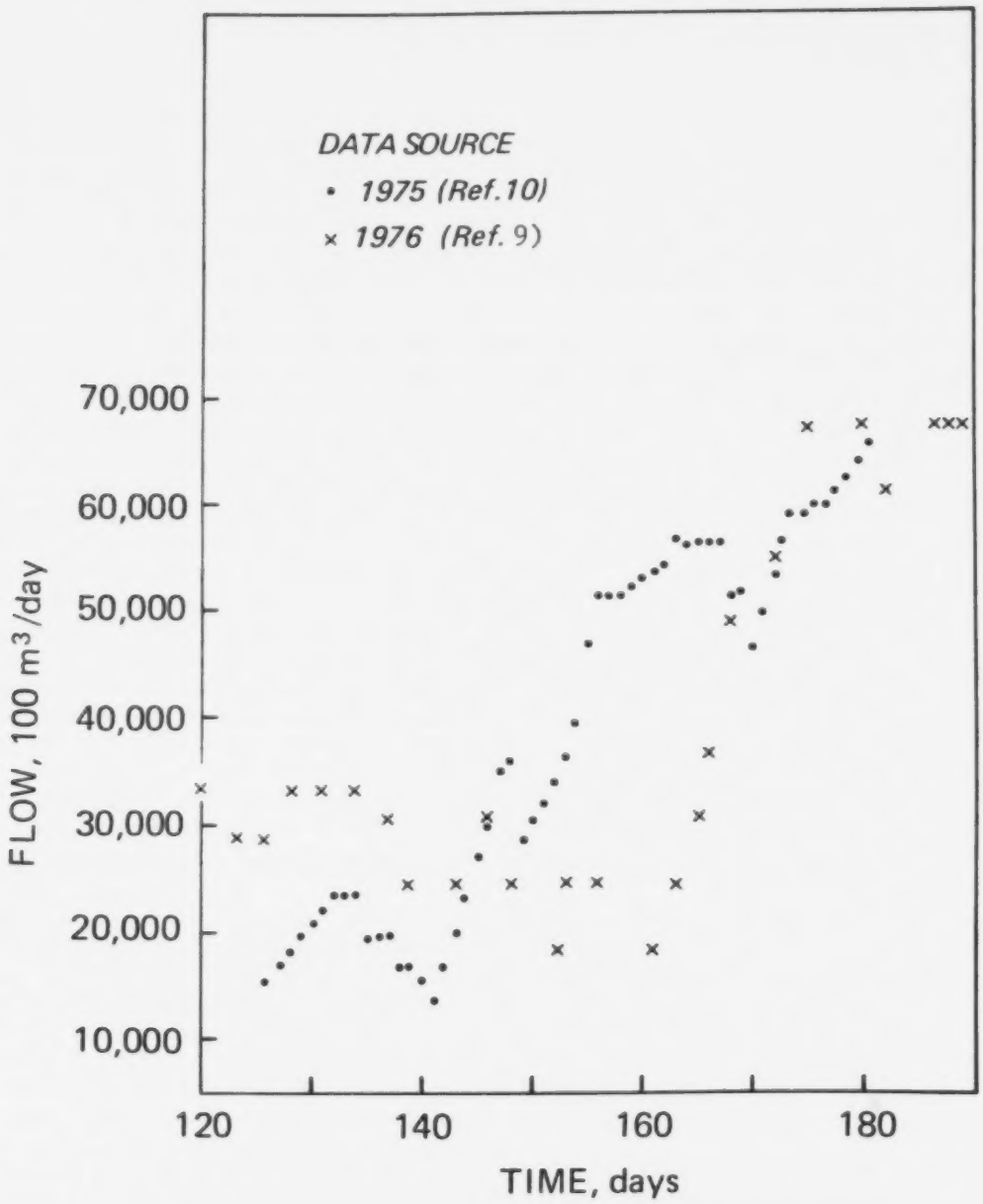

Figure 31. Daily cooling water pumping rate at Edison Plant Monroe, Michigan (May to July, 1975-76).
Data Source: 1975 - Ref.(10); 1976 - Ref.(9).

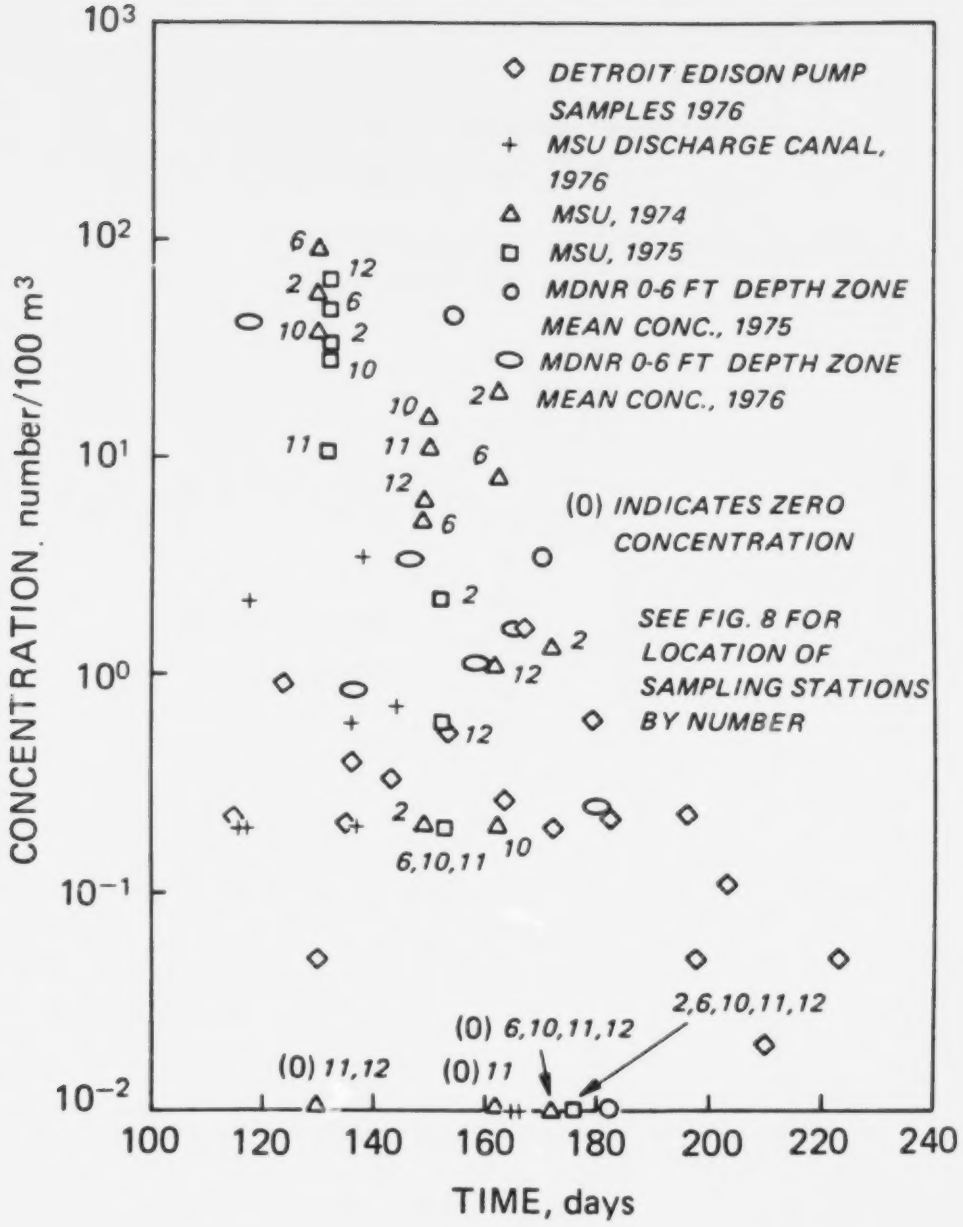

Figure 32. Larval perch concentration in vicinity of Monroe Plant cooling water intake (see Figure 8 for locations of sampling stations).

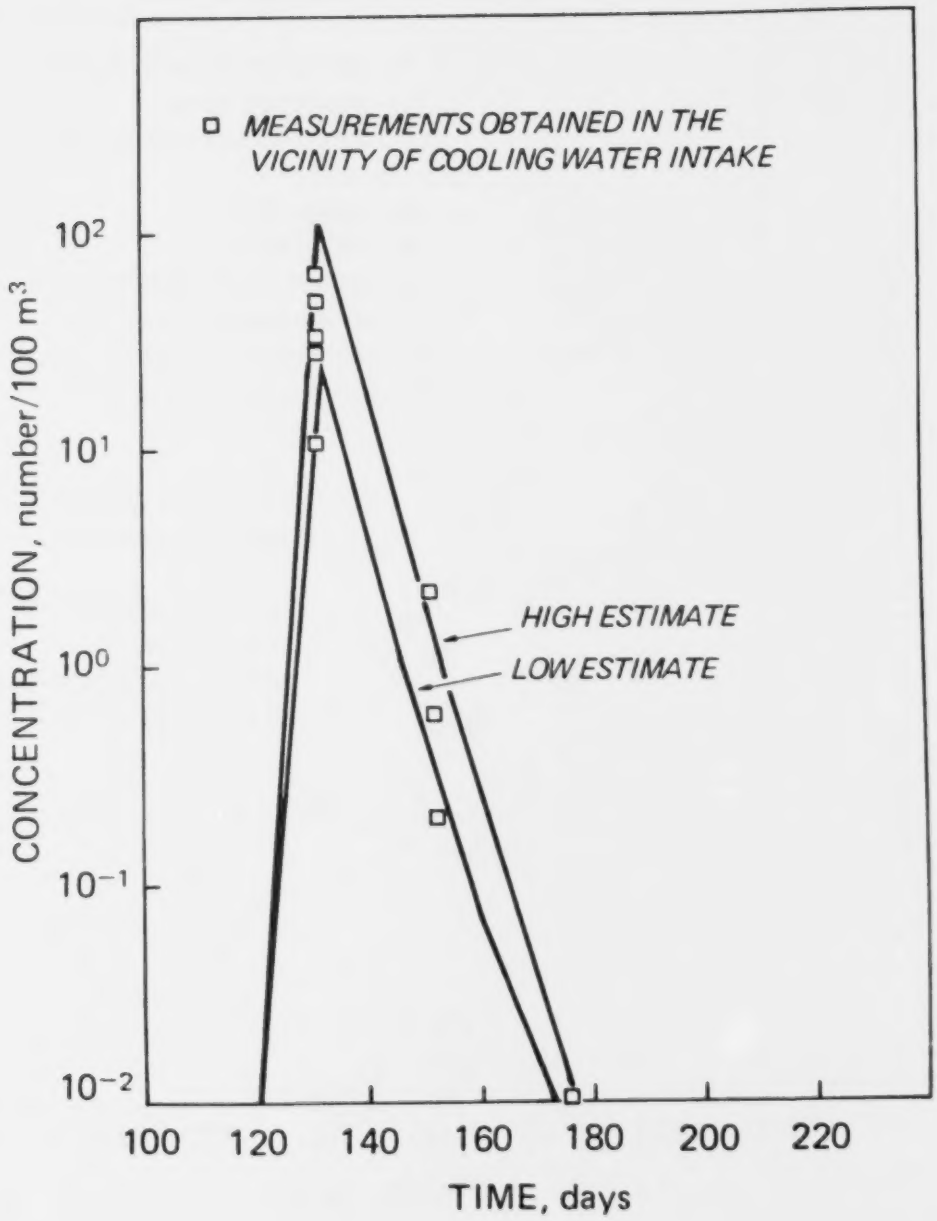

Figure 33. Larval perch concentrations estimated in Monroe Plant cooling water (1975).
Data Source: Ref.(4).

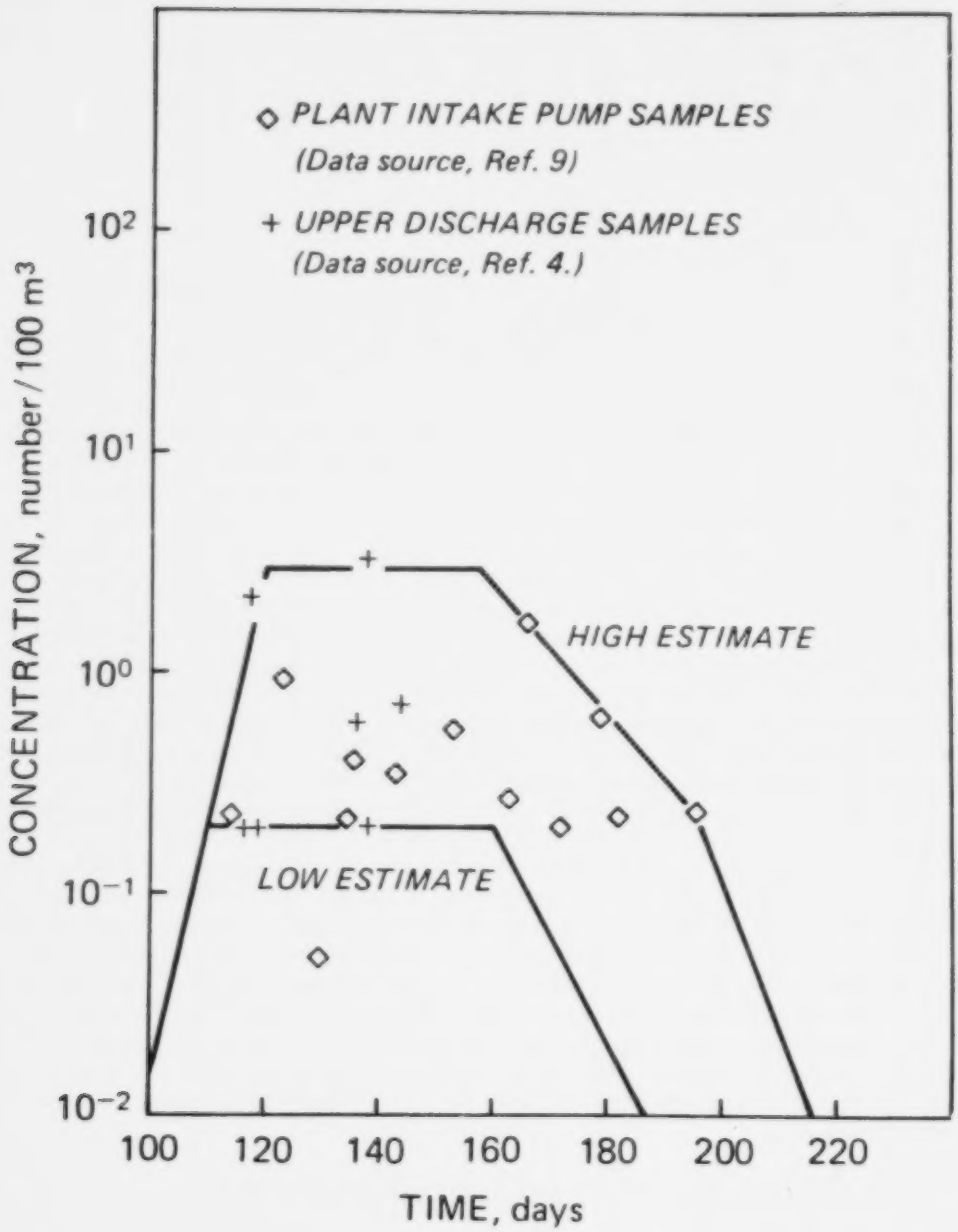

Figure 34. Larval perch concentrations estimated in Monroe Plant cooling water (1976).
Data Source: Ref.(4).

$$E(t) = \begin{cases} 0 & (0 < t \leq 125) \\ 134{,}000 & (125 < t \leq 132) \\ 265{,}764 & (132 < t \leq 141) \\ 126{,}500 & (141 < t \leq 148) \\ 66{,}361 & (148 < t \leq 156) \\ 21{,}209 & (156 < t \leq 170) \\ 0 & (170 < t \leq 365) \end{cases} \qquad (14)$$

Analysis of concentrations of larval yellow perch collected at the same stations over the same period in 1974 (Table 15) indicates that a larger number of larval perch may have been entrained in 1974. Using their own pump sampled data (9), Detroit Edison personnel estimated entrainment of larval yellow perch in 1976 at 650,000, a drop of nearly one order of magnitude from 1975. This estimate was checked in two different ways. First, the daily estimates of numbers of larvae entrained (calculated by Detroit Edison) were divided by daily volume of cooling water (Figure 32) to obtain estimates of mean concentrations of larvae in the cooling water column. These estimates were then compared to measurements that MSU obtained of concentrations of larval perch in the upper discharge (Figure 34). A statistical test revealed no significant difference in the mean values of the two sets of concentrations. A second method of checking the plausibility of the estimate of 650,000 perch larvae entrained in 1976 consists of comparing this figure to Detroit Edison's 1975 estimate, as a percentage of total production in Michigan waters. In 1975, an estimated total of 2.9×10^8 to 5.2×10^8 perch larvae were produced in Michigan waters. Detroit Edison estimated that 5.0×10^6 perch larvae or 1.0% to 1.7% of the estimated production in Michigan waters were entrained in 1975. In 1976, production declined to an estimated 8.4×10^7 to 1.4×10^7, so that the percentage of production estimated to have been entrained is 0.2% to 0.4%, about 22 percent of the 1975 percentage. This comparison suggests that Detroit Edison's estimate of number of larvae entrained in 1976 may be low. If percentage of larvae produced and entrained in 1976 were the same as in 1975, the estimated number entrained in 1976 would increase to 8.4×10^5 to 2.4×10^6. A combination of data from Figures 31 and 34 and the use of equation 13, yield an estimate of 195,000 to 2,827,000 killed due to the entrainment cycle.

Entrainment by Other Industrial and Municipal Water Intakes

A total of 18 municipal and industrial water intakes have been located in Michigan-Ohio waters of the western basin of Lake Erie (11). Estimates of numbers of yellow perch larvae entrained by all 18 intakes in 1975 to 1976 are reproduced in the present report as Tables 18 to 21 (11). A combination of the estimated mean daily pumping rates given in (11) with estimates of larval concentrations in the respective 0' to 6' depth zones revealed the following estimated total numbers of larval yellow perch losses

TABLE 13. WATER VOLUMES IN OHIO WATERS OF WESTERN BASIN.
(From Ref. 5) (cubic meters)

Sector \ Depth Zone	1	2	3	4	5	6	Total
A	5.73×10^{7}	9.3×10^{7}	1.71×10^{8}	0	0	0	3.21×10^{8}
B	0	0	0	7.77×10^{8}	9.72×10^{8}	1.06×10^{9}	3.21×10^{9}
C	2.17×10^{7}	2.44×10^{8}	6.3×10^{8}	7.63×10^{8}	4.73×10^{8}	0	2.13×10^{9}
D	7.15×10^{7}	2.41×10^{7}	6.10×10^{7}	2.48×10^{8}	8.29×10^{8}	6.38×10^{8}	1.81×10^{9}
E	7.02×10^{6}	2.94×10^{7}	1.17×10^{8}	2.60×10^{8}	3.57×10^{8}	1.51×10^{9}	2.28×10^{9}
F	4.28×10^{6}	2.26×10^{7}	1.01×10^{8}	3.46×10^{8}	6.53×10^{8}	1.71×10^{9}	2.84×10^{9}
TOTAL	9.745×10^{7}	4.131×10^{8}	1.08×10^{9}	2.394×10^{9}	3.284×10^{9}	4.918×10^{9}	1.22×10^{10}

TABLE 14. CONCENTRATIONS OF LARVAL YELLOW PERCH AT STATION 2 IN CANADIAN WATERS.
Data Source: Ref. (2,3)

Date	Concentration (#/100 M^3) Bottom	Surface
1976		
5-17-76	11.82	-
5-25-76	8.56	6.59
6-8-76	-	0.78
7-9-76	0.74	-
7-21-76	1.86	-
1975		
6-18-75	0.64	0

TABLE 15. YELLOW PERCH LARVAL CONCENTRATIONS SAMPLED IN IMMEDIATE VICINITY OF POWER PLANT.
Data Source: Table B-26, Vol. II, Ref. (4).

Date	Station Number 6	10	11	12	2
5-10-74	90.8	37.6	0	0	57.6
5-29-74	5.0	15.3	10.7	6.2	0.2
6-11-74	8.0	0.2	0	1.1	20.8
6-21-74	0	0	0	0	1.3
5-12-75	48.1	28.0	10.5	65.5	33.4
6-2-75	0.2	0.2	0.2	0.6	2.2
6-25-75	0	0	0	0	0

TABLE 16. COEFFICIENTS OF SAMPLING VARIATION ASSOCIATED WITH MEAN CONCENTRATIONS.
Data Source: Table B-31, Vol. II, Ref. (4).

Date	Station Number 6	10	11	12	2
5-29-74	43.8	39.4	45.5	64.1	78.1
6-11-74	47.0	244.9	0	164.7	34.7
5-12-75	110.1	63.7	43.9	39.0	49.3
6-2-75	306.2	244.9	113.9	173.3	95.6

TABLE 17. ESTIMATED NUMBER OF YELLOW PERCH LARVAE ENTRAINED BY MONROE POWER PLANT IN 1976.
Data Source: Ref. (9)

Date	Estimated Number Entrained (24 hr)	Flow 100 M^3/day	Mean Concentration Larv. #/100 M^3
108-114	45,488	36,469	0.21
115-120	36,645	33,445	0.22
121-127	161,831	28,892	0.93
128-134	10,364	33,445	0.05
135			0.22
136	9,768	24,434	0.40
137	10,219	30,552	0.33
139	3,987	24,434	0.16
143	8,372	24,434	0.34
144	1,395	24,434	0.06
146	2,991	30,552	0.10
148	0	24,434	0
152	7,471	18,316	0.41
153	13,356	24,434	0.55
154	12,545	24,434	0.51
156	15,947	24,434	0.65
158	5,183	24,434	0.21
159	997	24,434	0.04
161	2,391	18,316	0.13
162	2,813	18,316	0.15
163	6,578	24,434	0.27
164	3,940	24,434	0.16
165	5,982	30,552	0.20

TABLE 17 (CONTINUED)

Date	Estimated Number Entrained (24 hr)	Flow 100 M^3/day	Mean Concentration Larv. #/100 M^3
166	60,668	36,632	1.66
168	30,699	48,868	0.63
172	10,759	54,948	0.20
173	1,913	54,948	0.03
175	20,672	67,184	0.31
179	41,825	67,184	0.62
180	14,408	67,184	0.21
182	13,285	61,066	0.22
187	3,898	67,184	0.06
188	12,278	67,184	0.18
189	5,847	67,184	0.09
183	4,722	67,184	0.07
194	3,508	67,184	0.05
196	15,476	67,184	0.23
197	5,977	73,264	0.08
198	3,586	73,264	0.05
199	7,876	73,264	0.11
200	1,793	73,264	0.02
201	4,782	73,264	0.06
203	8,368	73,264	0.11
207	1,195	73,264	0.02
208	1,275	73,264	0.02
210	1,275	73,264	0.02
213	1,195	73,264	0.02
214	0		0
216	1,195	73,264	0.02
220	0		0
221	0		0

TABLE 17 (CONTINUED)

Date	Estimated Number Entrained (24 hr)	Flow 100 M^3/day	Mean Concentration Larv. #/100 M^3
223	2,953	54,948	0.05
224	0		0
225	0		0
226	0		0
227	0		0
228	0		0
	649,591		

Estimates based upon Detroit Edison data on estimated number entrained per day and flow rates.

TABLE 18. WATER INTAKE SPECIFICATIONS.
Data Source: Ref. (11)

Intake	Lake Sector	Depth Zone		Pumping Rate (100 M^3/ day)
Michigan				
Fermi (P)	M	1-2		9274
Monroe (P)	M	1-2		78299
Whiting (P)	M	1-2		11671
Monroe City	M	3-4		303
			SUBTOTAL	99547
Ohio				
Acme (P)	R	R		14716
Bayshore (P)	R	R		28342
Davis-Besse (P)	C	2		818
Camp Perry	C	2		9.5
East Harbor State Park	D	3		3
Erie Industrial Park	C	2		8
Kelleys Island	E	2		3
Lakeside Association	E	2		8
Marblehead	E	2		4.5
Oregon	A	3		160
Port Clinton	C	2		57
Put-In-Bay	D	2		5
Sandusky	E	4		404
Toledo	A	3		303
			SUBTOTAL	44841
			TOTAL	144388

(P) - Power Plant

TABLE 19. LARVAE ENTRAINMENT ESTIMATES, 1975.
Data Source: Ref. (11)

Intake	Entrainment Estimate	
	Point Sample	Depth Zone
Michigan		
Fermi (P)	61,000	349,000
Monroe (P)	531,000	2,940,000
Whiting (P)	268,000	439,000
Monroe City	2,100	69,200
SUBTOTAL	862,700	3,797,200
Ohio		
Acme (P)*	-	2,340,000
Bayshore (P)	1,686,300	4,510,000
Davis-Besse (P)**	-	-
Camp Perry	9,000	14,800
East Harbor State Park	200	900
Erie Industrial Park	7,200	11,900
Kelleys Island	900	400
Lakeside Association	700	1,000
Marblehead	400	600
Oregon	2,800	6,900
Port Clinton	1,200	88,700
Put-In-Bay	1,500	300
Sandusky	2,600	61,500
Toledo	50,900	124,000
SUBTOTAL	1,763,700	7,161,000
TOTAL	2,626,400	10,958,200

(P) - Power Plant

* - No fish caught at sampling station in 1975.
** - Davis-Besse not operating in 1975.

TABLE 20. LARVAE ENTRAINMENT ESTIMATES, 1976.
Data Source: Ref. (11)

Intake	Entrainment Estimate	
	Point Sample	Depth Zone
Michigan		
Fermi (P)	265,300	728,000
Monroe (P)	1,625,700	6,150,000
Whiting (P)	1,520,600	917,000
Monroe City	6,300	544,600
SUBTOTAL	3,417,900	8,339,600
Ohio		
Acme (P)*	-	24,200,000
Bayshore (P)	1,181,400	46,600,000
Davis-Besse (P)	17,200	344,000
Camp Perry	12,300	3,900
East Harbor State Park	700	400
Erie Industrial Park	9,600	3,100
Kelleys Island	400	300
Lakeside Association	2,300	900
Marblehead	2,100	500
Oregon	6,600	6,200
Port Clinton	47,500	23,200
Put-In-Bay	300	29,200
Sandusky	94,200	270,100
Toledo	118,400	112,200
SUBTOTAL	1,493,000	71,584,000
TOTAL	4,910,900	79,923,600

(P) - Power Plant

* - No fish caught at sampling station in 1976.

TABLE 21. RANGES OF ENTRAINMENT LOSSES

Intake	1975	1976
Michigan		
Fermi (P)	61,000-349,000	265,000-728,000
Monroe (P)	531,000-2,940,000	1,636,000-6,150,000
Whiting (P)	268,600-439,000	917,000-1,521,000
Monroe City	2,100-69,200	6,300-544,600
Ohio		
Acme (P)	0-2,340,000	0-24,200,000
Bayshore (P)	1,690,000-4,510,000	1,180,000-46,600,000
Davis-Besse (P)*	-	17,200-334,000
Camp Perry	9,000-14,800	3,900-12,300
East Harbor State Park	200-900	400-700
Erie Industrial Park	7,200-11,900	3,100-9,600
Kelleys Island	300-1,000	300-400
Lakeside Association	700-1,000	900-2,300
Marblehead	400-600	500-2,100
Oregon	2,800-6,900	6,200-6,600
Port Clinton	1,200-88,700	4,700-23,200
Put-In-Bay	300-1,500	300-2,900
Sandusky	2,600-61,000	94,000-270,000
Toledo	50,800-124,000	112,000-118,000

(P) - Power Plant

* - Davis-Besse not in operation during 1975.

Range of larvae entrainment by Michigan and Ohio water intakes. Data based upon both point sample and depth zone entrainment.

attributable to all power plant operations in Michigan-Ohio waters of the western basin:

Intake	1975	1976
Michigan		
Fermi	48,000-1,100,000	261,000-3,300,000
Monroe	1,432,000-9,833,000	195,000-2,827,000
Whiting	827,000-1,525,000	74,000-1,251,000
Ohio		
Acme	497,000-1,700,000	520,000-1,363,000
Bayshore	879,000-2,500,000	733,000-1,850,000
Davis-Besse	- -	17,000-334,000
TOTAL	3,683,000-16,658,000	1,800,000-10,925,000

Since Tables 18 to 21 and the above estimates of total losses attributable to all power plant operations became available after the numerical analysis of production was completed, the assumption made for purposes of the analyses is:

$$L(t) = 0 \qquad (15)$$

Overall estimates of production can be adjusted by adding estimated losses due to entrainment in water intakes.

Analytical Solution to the Differential Equation of Balance for Larval Concentration

The equation of balance for larval perch assumes the form

$$\dot{N}(t) + p \cdot N(t) = h(t) - h(t-25) \cdot e^{-25p} - E(t) \qquad (16)$$

with the substitution of equations (4), (5), (11), (12) (14) and (15) into equation (3). The expressions for h(t) and E(t) depend upon the reference volume and the year being considered. Solutions to Equation (16) for five cases - Ohio 1975 and 1976, Michigan 1975 with and without entrainment mortality, and Michigan 1976 - are given in Appendix F. Equations (F.7), (F.9), (F.11), (F.13), and (F.15) were programmed permitting the parameters h and p to range over assigned values as shown in Figures 35, 37, 40, and 42. Specific solutions as illustrated in Figures 36, 38, 41, and 43 are obtained for each specific (h, p) combination.

Method of Estimating Parameters h and p

The parameters h (number of perch larvae added to every 100 M^3 of water in the reference volume in a given year), and p (mean daily natural mortality rate of perch larvae in the reference volume in a given year) are estimated by the method of least squares. For a given combination of h and p, the predicted value of larval concentration (number of larvae per 100

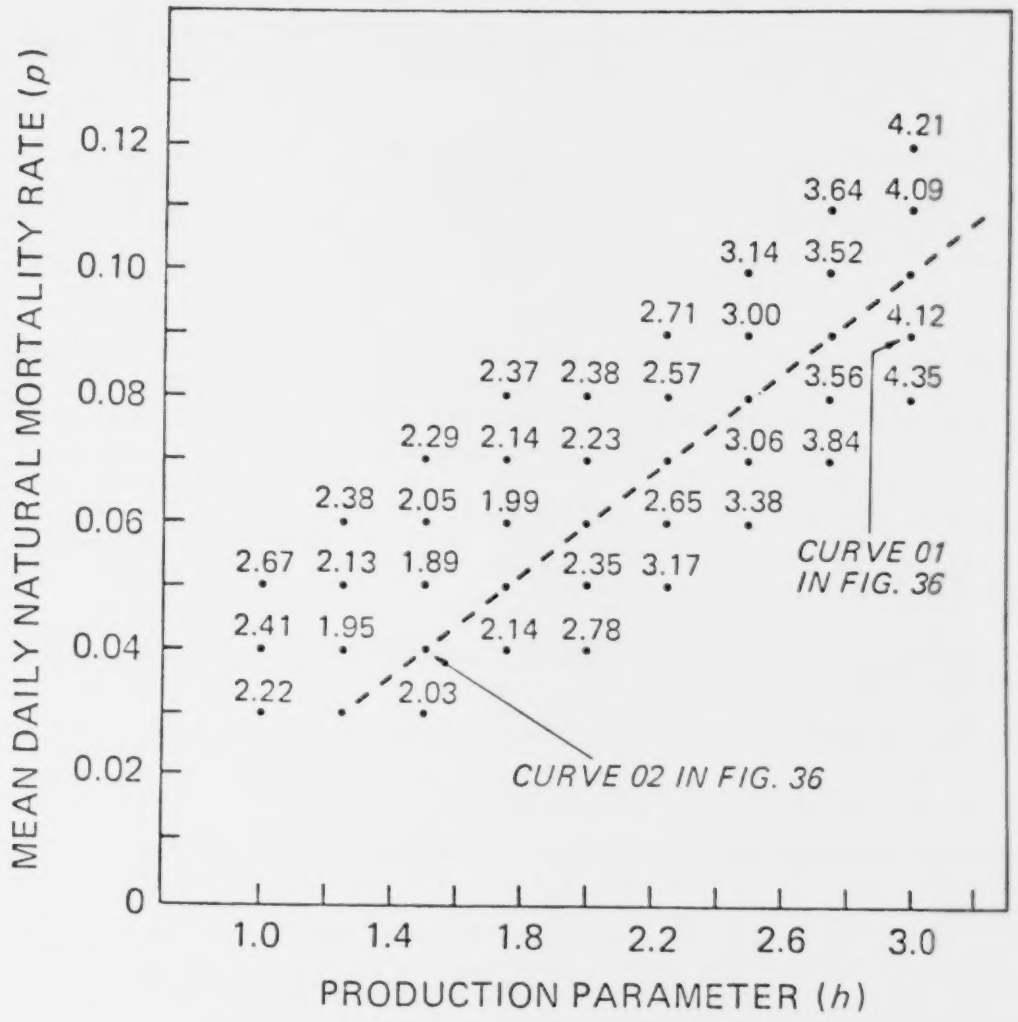

Figure 35. Model prediction error for combinations of mortality and production parameters (Ohio, 1975).

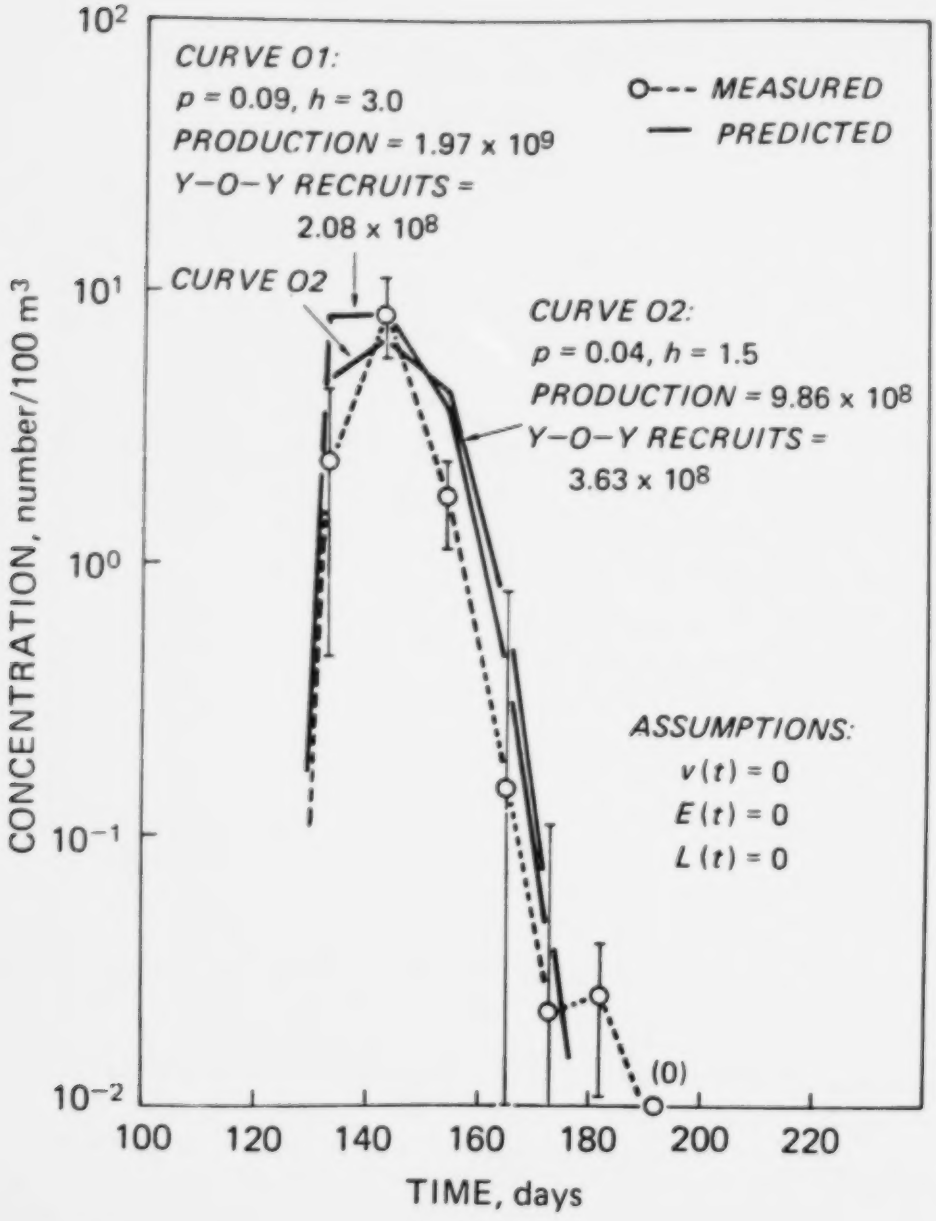

Figure 36. Predicted vs. estimated larval perch concentrations for two production - mortality parameter combinations (Ohio, 1975).
Data Source: Tables 12A and 12B.

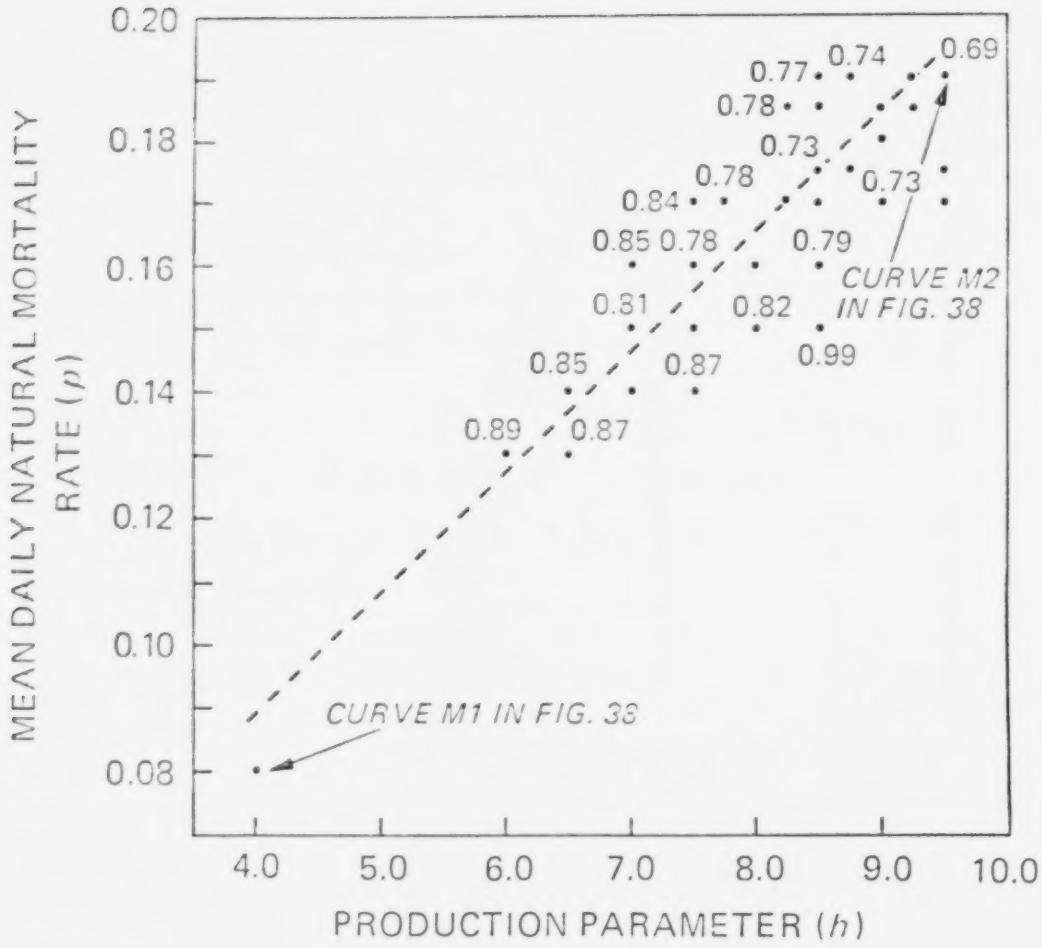

Figure 37. Model prediction error for combinations of mortality and production parameters (Michigan, 1975).

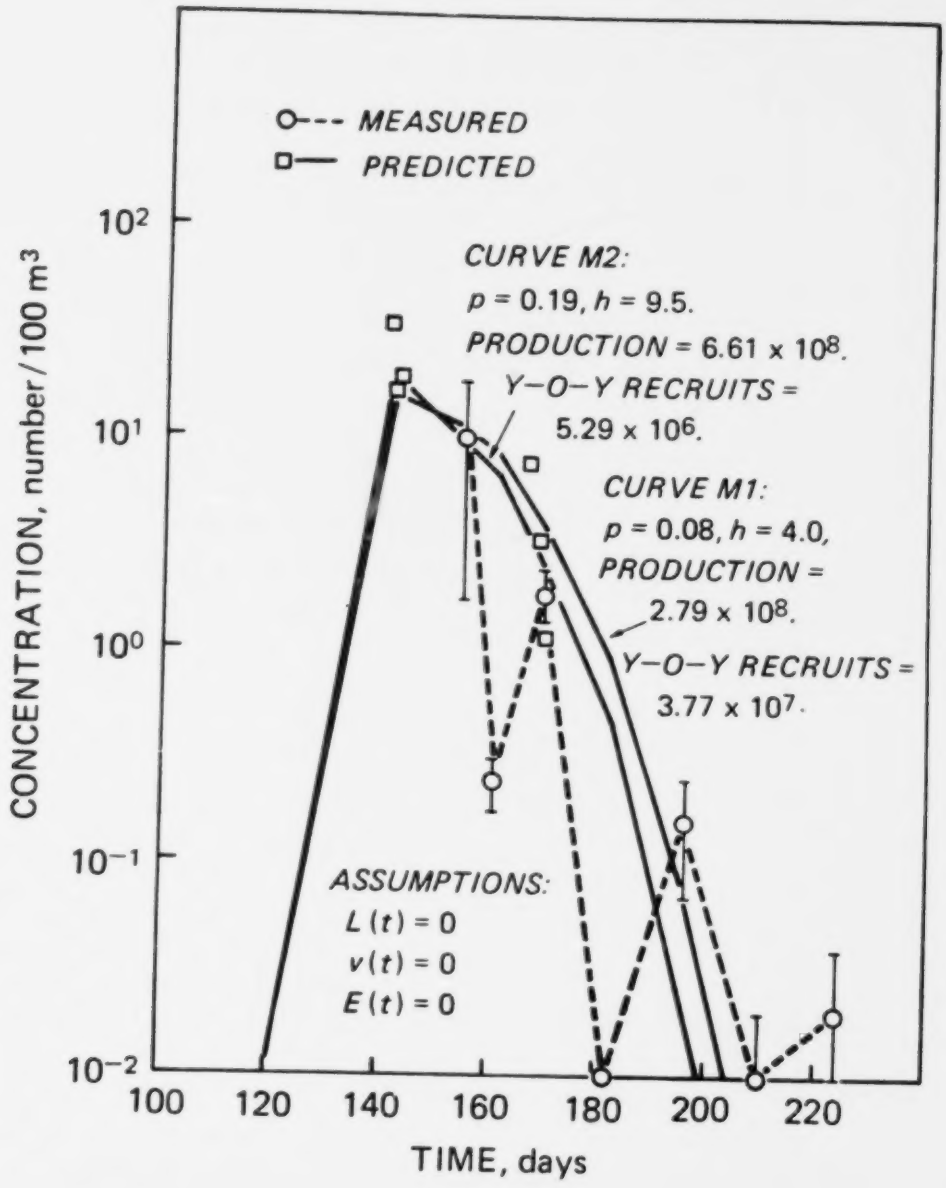

Figure 38. Predicted vs. estimated larval perch concentrations for two production - mortality parameter combinations (Michigan, 1975).
Data Source: Table 1.

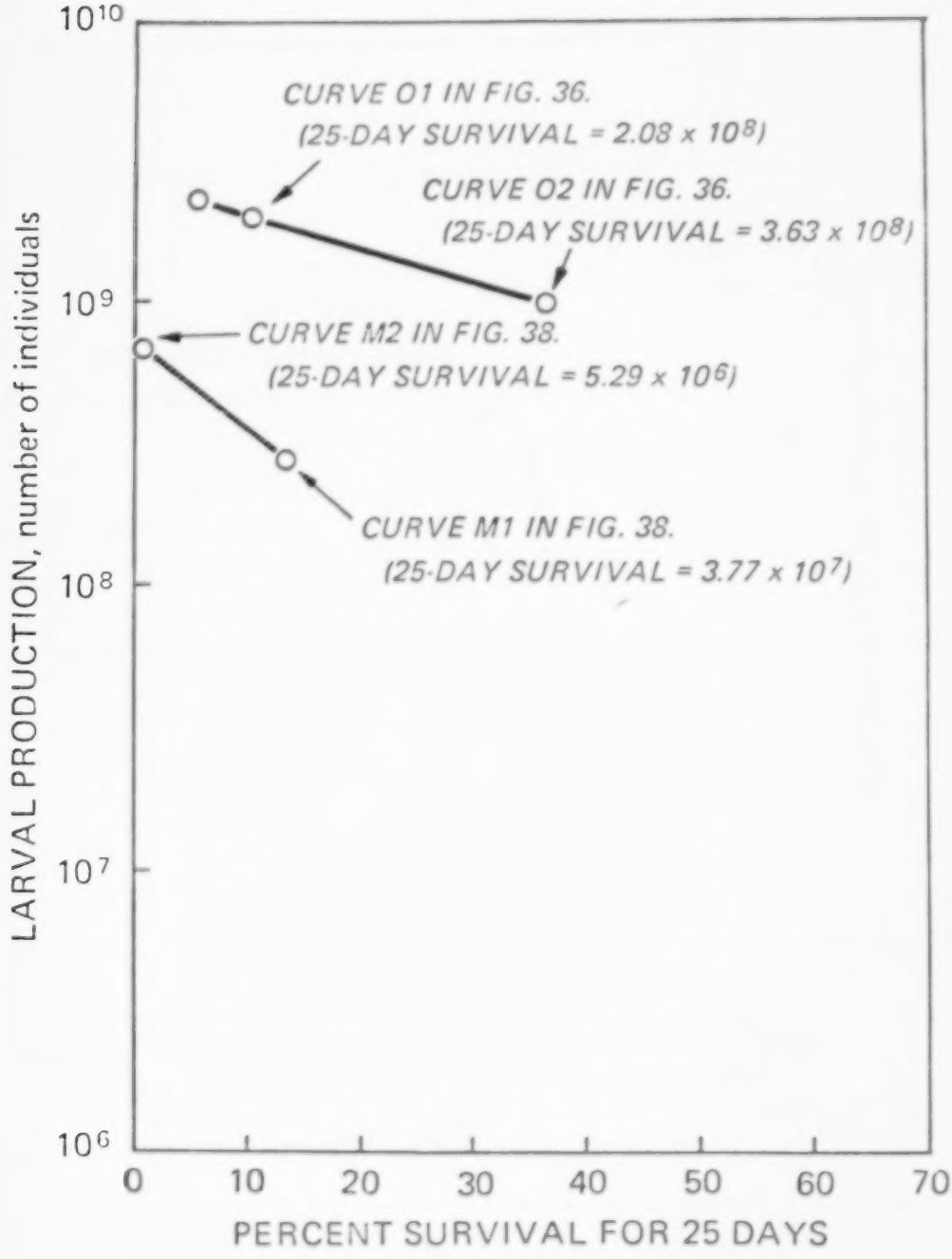

Figure 39. Plausible larval perch production - survival combinations in Western Basin (1975).

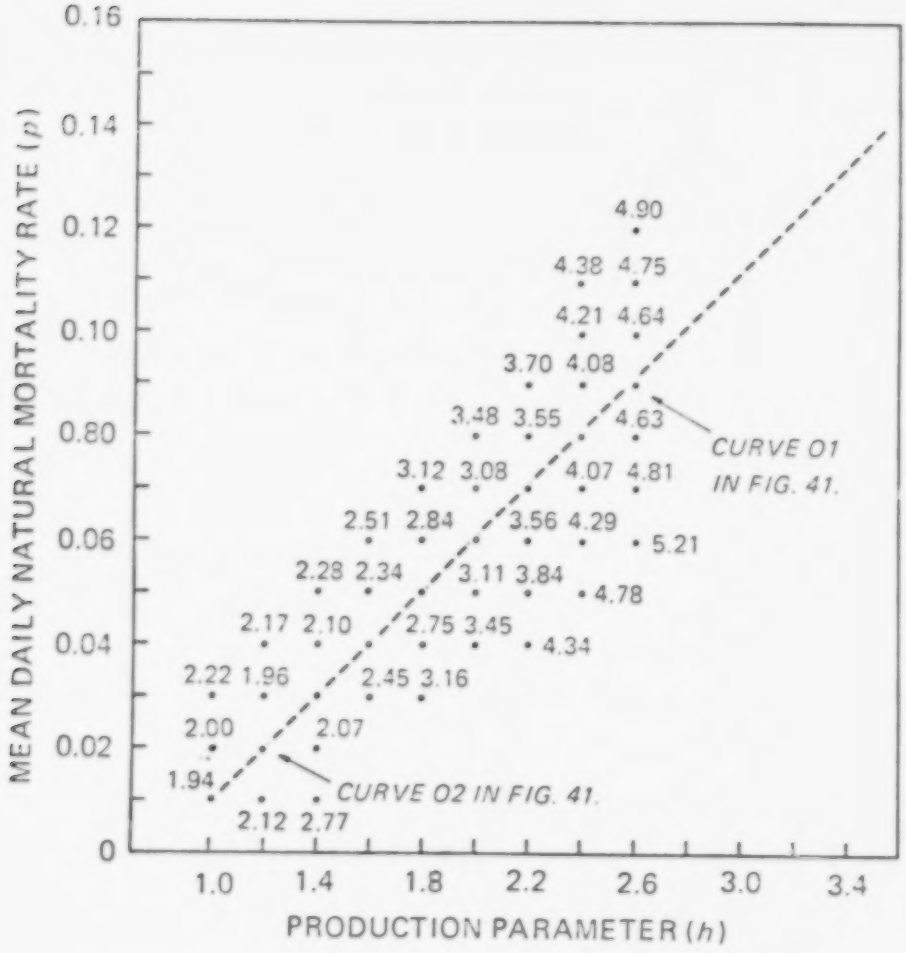

Figure 40. Model prediction error for combinations of mortality and production parameters (Ohio, 1976).

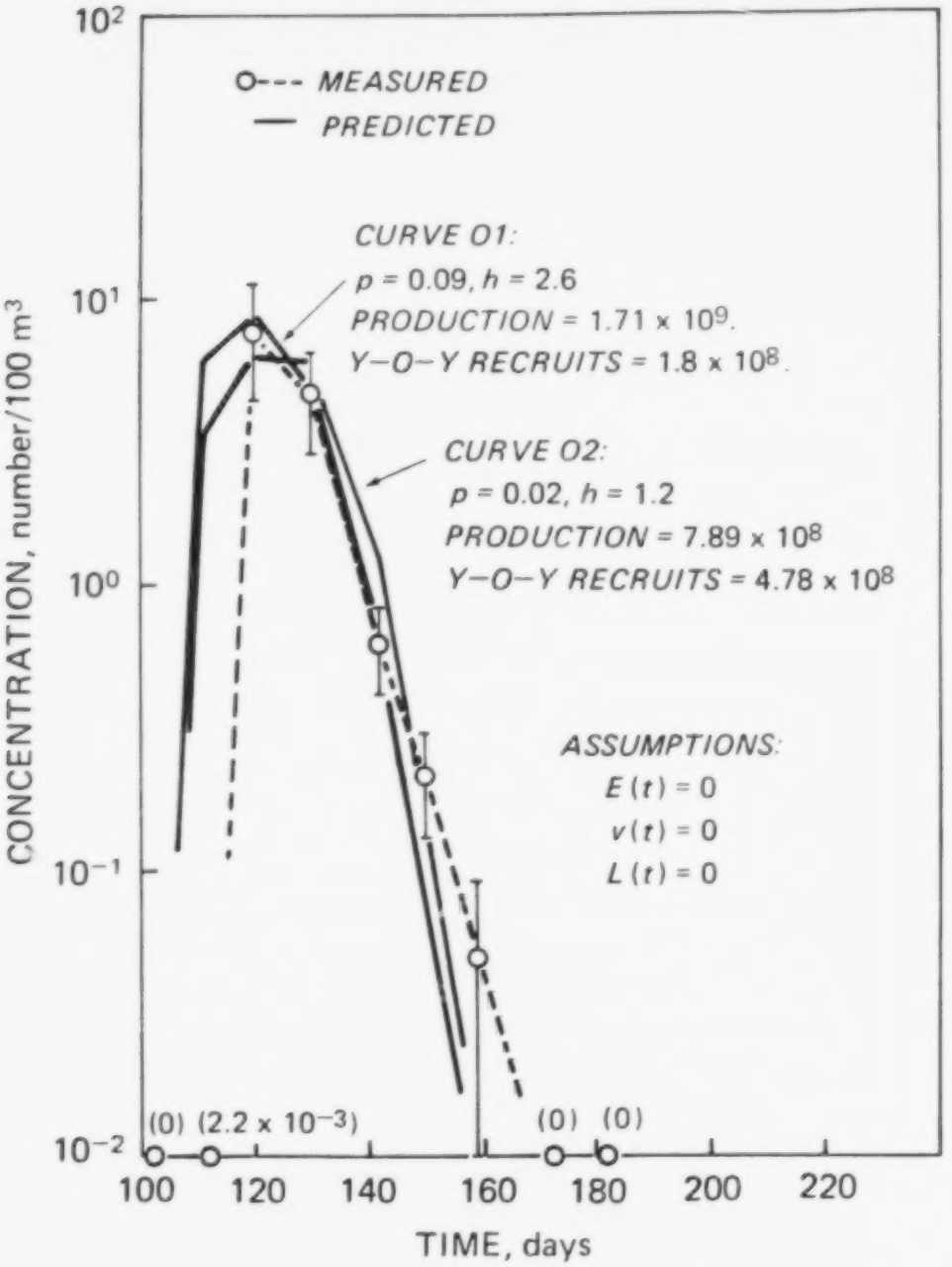

Figure 41. Predicted vs. estimated larval perch concentrations for two production - mortality parameter combinations (Ohio, 1976).
Data Source: Tables 12C and 12D.

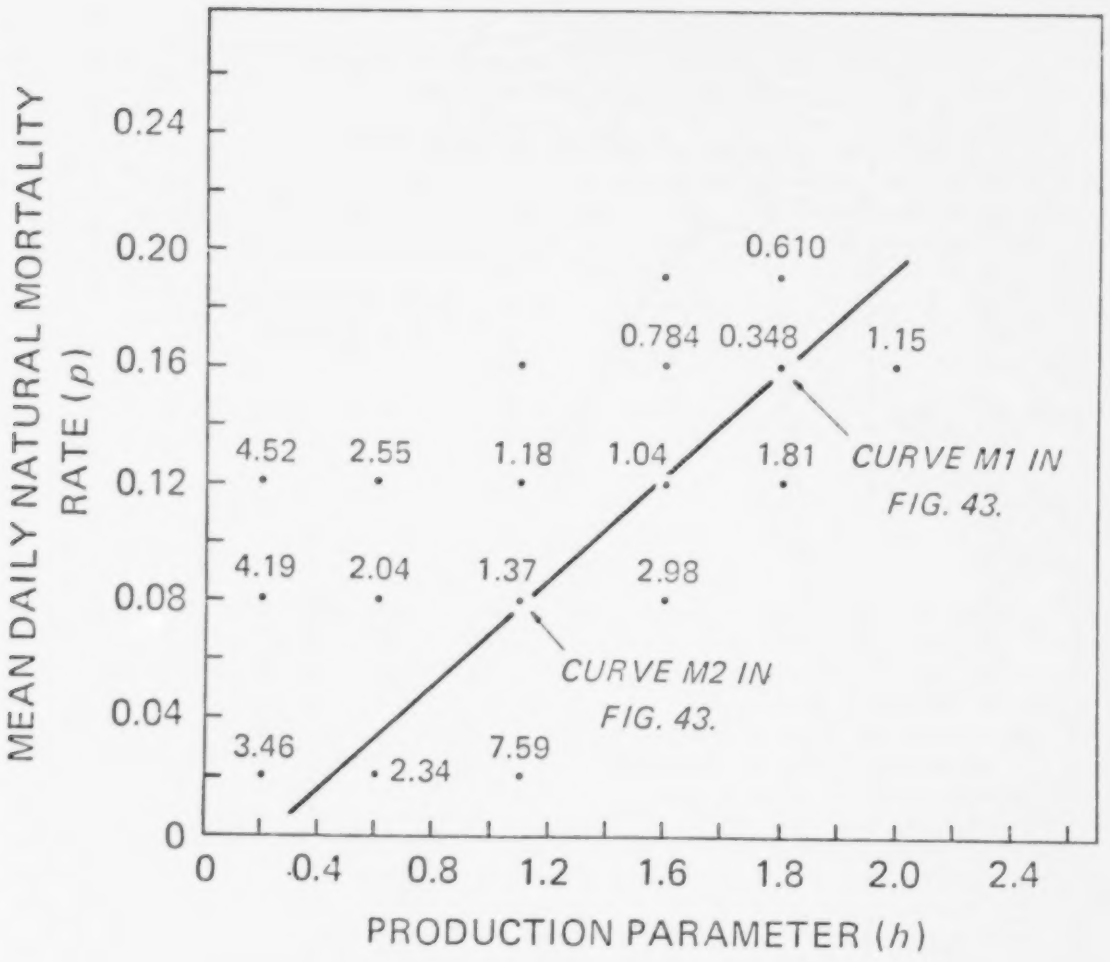

Figure 42. Model prediction error for combinations of mortality and production parameters (Michigan, 1976).

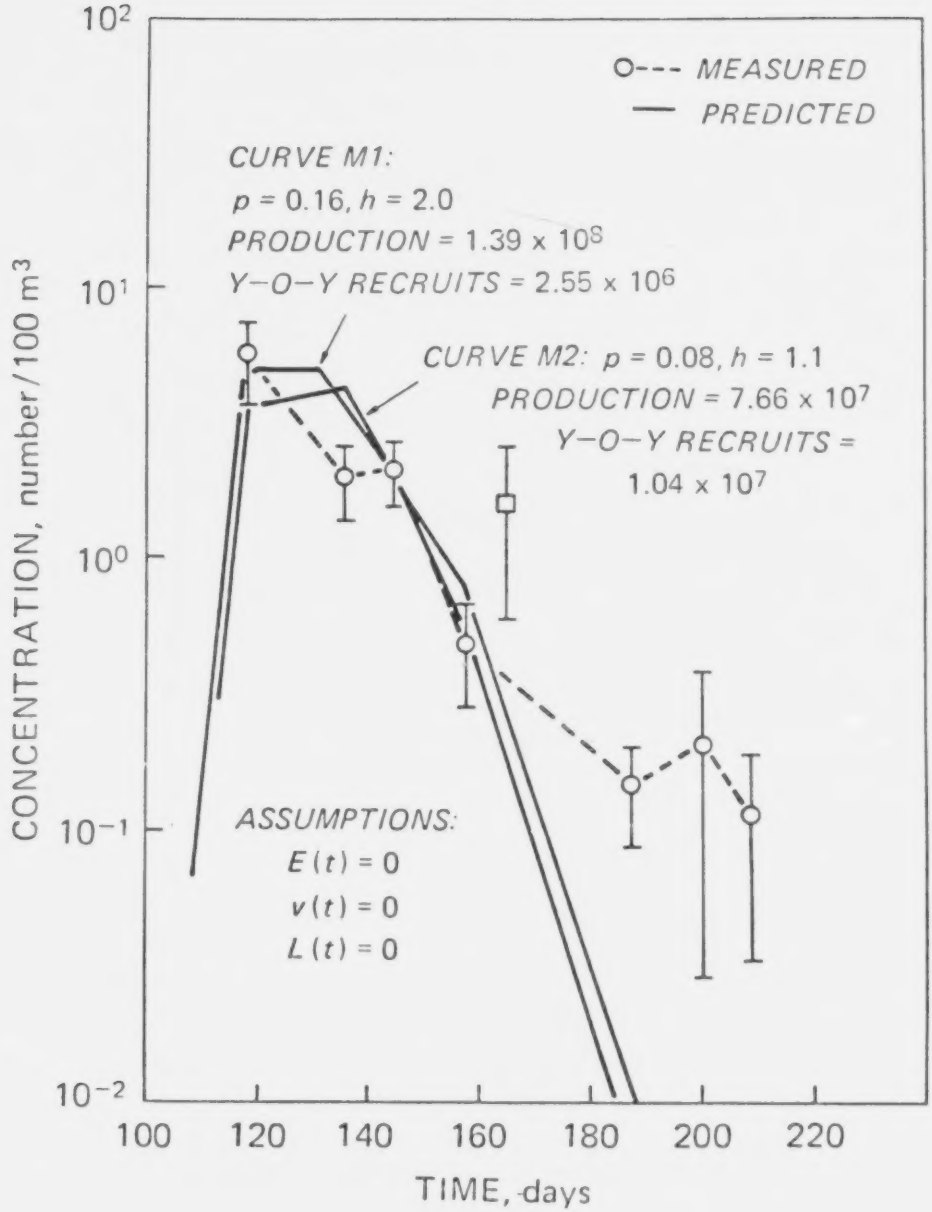

Figure 43. Predicted vs. estimated larval perch concentrations for two production - mortality parameter combinations (Michigan, 1976).
Data Source: Table 9.

cubic meters of water in the reference volume at a given time) provided by the solution to Equation (3) is compared to a mean concentration estimated from field data analysis (plotted in Figures 7, 13, 15, and 16). The Mean Square Error, M.S.E.(h,p), is by definition:

$$\text{M.S.E.}(h,p) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{1}{B}N(t_i) - \text{estimated mean conc. on day } t_i\right)^2 \quad (17)$$

Following the least squares criterion, the combinations of h and p which minimize the M.S.E. (for a given reference volume and year) are shown in Figures 35, 37, 40, and 42. If either $\hat{h}$ or $\hat{p}$ is selected in advance, the value of the other that minimizes M.S.E. can be obtained from the appropriate figure. If h and p are two values selected by minimizing Mean Square Error or MSE in a given case, then from Equation (6), total larval production and 25-day survival for the given reference volume and year is estimated as:

$$\text{Total Production} = d \cdot B \cdot \hat{h} \quad (18)$$

and:

$$\text{number of y-o-y recruits} = \text{25 day survival} = d \cdot B \cdot \hat{h} \cdot e^{-25 \cdot \hat{p}} \quad (19)$$

For a given reference volume and year, estimation of h and p proceeds by defining a rectangular network of (h,p) pairs. The prediction error variance is numerically evaluated for selected (h,p) combinations and recorded as shown in Figures 35, 37, 40, and 42. The finer the mesh of the grid (the closer together the (h,p) combinations) the more precisely the parameter combinations that minimize prediction error variance can be estimated. For example, in Figures 35 and 40 the h-axis is graduated in increments of 0.2; this graduation corresponds to an increase in total larval production in the reference volume of (0.2) (7) (9.393×10^7) = 1.31×10^8 larvae. Therefore, any term on the right hand side of equation (16) that is less than 10 percent of 1.31×10^8, or about 13 million larvae, is not likely to produce any difference in the pair (h,p) that minimizes M.S.E. The broken lines shown in Figures 35, 37, and 40 show the values of p that approximately minimize M.S.E. for given values of the production parameter h. It was initially anticipated that a unique global optimum pair (h,p) would be identified for each case analyzed. Such optima are shown for Ohio waters: h = 1.5, p = 0.04 for Ohio 1975, and h = 1.2, p = 0.02 for Ohio 1976. However, a value of p = 0.04 corresponds to a 25-day survival (y-o-y recruitment) of 36.8 percent, a value considered to be too high, that is, biologically unrealistic. In Michigan waters: the 1975 combination of (h,p) that minimized M.S.E. is located on the boundary of the grid (h = 9.5, p = .19); for Michigan 1976 the optimum occurs at an interior point of the grid (h = 2.0, p = .16). A value of p = .19 corresponds to a 25-day survival of 0.9 percent and p = .16 corresponds to 1.8 percent survival for

25 days. These survival percentages may well be too low from a biological standpoint; they also reflect the lumping of emigration and water intake losses, L(t), into natural mortality. Overall 25-day survival is judged to be in the 2 to 10 percent range. It is clear from the analysis that production in Ohio waters is much greater than in Michigan waters. It also appears that changes in production on the order of 15 to 20 percent from one year to the next are detectable. It is reported in (12) that October 1976 trawls in the western basin declined seven-fold in young-of-year perch abundance from October 1975. Comparison of Figures 35 and 40 suggest that larval production may have declined in Ohio waters from 1975 to 1976 (peak mean abundance was lower in 1976), but based upon this study, it is improbable that a seven-fold drop in 25 day larval survival occurred from 1975 to 1976. If y-o-y recruitment did, in fact, decrease seven-fold, it may be hypothesized yellow perch year class strength is heavily influenced during the late post larval phase of development. The broken lines in Figures 35, 37, and 40 which mark the M.S.E. estimates of p for fixed values of h should not be interpreted as defining relationships between production and 25-day survival because: a) each line is based upon data collected in a single year only, and b) the slopes are so steep that net 25-day survival actually decreases as production increases.

Modeling error can affect the locations of (h,p) pairs which minimize prediction error variance. Emigration, v(t), could be on the order of 5 to 10 percent of production and that entrainment mortality, L(t), from water intakes is estimated to be in the tens of millions. Both v(t) and L(t) were assumed to be zero for the computer runs described in this report. If L(t) and v(t) are programmed as positive functions, the resulting solution to equation (16) of larval balance will show a slighty improvement in the fit of the model for Ohio 1975 (Figure 36). It is not clear by simple inspection of the graphs of the other solutions (Figures 38, 41, and 43) whether increasing L(t) and v(t) will cause the predicted concentrations to fit the estimated concentrations more closely. In any case, a first order correction to estimates of production can be made by assuming $L(t) = v(t) = 0$, and by adding the estimates of total water intake mortality and total emigration to the estimates of production for each case. Two cases were analyzed for Michigan 1975 waters: $E(t) = 0$ (Figure 38), and E(t) specified by equation (14). Whereas small differences in the prediction error variance were observed between the two cases, the numerical values of E(t) were so small relative to the interval length on the h-axis (Figure 37) that no detectable differences for the locations of M.S.E. values of p were observed. A first order correction to production can be obtained by adding total estimated water intake mortality and total estimated emigration to previously estimated values of total production. Correcting biases in estimates of production caused by modeling errors (by assuming that $L(t) = E(t) = 0$) is somewhat academic inasmuch as total larval production can only be estimated to within tens of millions for Michigan waters and hundreds of millions for Ohio waters. However, if larval emigration out of the reference volume is as much as ten percent of total production, the correction could be as great as 2×10^8 larvae.

Estimates of Production and Natural Mortality

Single point estimates of production and natural mortality obtained by locating those (h,p) combinations that minimize prediction error variance can lead to estimates that are unrealistic due to a combination of errors outlined in Diagram A (p. 8). On the basis of the above method, however, larval survival over a 25-day period following hatching has been estimated to be between 2 and 10 percent. The range of larval production for which a 25-day survival of 2 to 10 percent is optimum in the M.S.E. sense is obtained from Figures 35, 37, 40, and 42 and is the following:

Michigan Waters, 1975

% Surviving 25 days:	2%	10%
Estimated value of h:	7.5	4.2
Estimated production:	5.2×10^8	2.9×10^8
Estimated number surviving natural mortality 25 days:	1.0×10^7	2.9×10^7
% (killed due to entrainment on 5th day of life) estimated to have otherwise survived 25 days:	5.5	17.7

Michigan Waters, 1976

% Surviving 25 days:	2%	10%
Estimated value of h:	2.0	1.2
Estimated production:	1.4×10^8	8.4×10^7
Estimated number surviving natural mortality 25 days:	2.8×10^6	8.4×10^6
% (killed due to entrainment on 5th day of life) estimated to have otherwise survived 25 days:	5.5	17.7

Ohio Waters, 1975

% Surviving 25 days:	2%	10%
Estimated value of h:	4.5	3.0
Estimated production:	3.0×10^9	2.0×10^9
Estimated number surviving after 25 days:	6.0×10^7	2.0×10^8

Ohio Waters, 1976

% Surviving 25 days:	2%	10%
Estimated value of h:	3.8	2.6
Estimated production:	2.5×10^9	1.7×10^9
Estimated number surviving after 25 days:	5.0×10^7	1.7×10^8

Estimates of production for other 25-day survival percentages can be obtained from Figures 39 and 44. It is clear from the above that recruitment into the young-of-year class is more sensitive to the natural mortality rate than to the number of eggs that are hatched in a given year. When the M.S.E. criterion is used to match mortality and production rates this point is clearly illustrated in Figures 35, 37, 40, and 42. If larval production is high, the use of M.S.E. estimate of the mean daily natural mortality rate results in lower numbers for recruitment into the young-of-year class than for cases where larval production is lower. It may be argued that a realistic relationship between larval production and young-of-year recruitment requires that marginal recruitment into the young-of-year class must be a non-negative function of larval production. It is pointed out that the present analysis does not deal with this question but only with the estimation of larval production and natural mortality for the years 1975 to 1976. The percent loss in recruitment into the young-of-year class attributable to entrainment mortality at the Monroe power plant is a more realistic measure of impact than percent loss of larval production because it takes into account natural mortality of larvae as well as larval production.

Estimated percent loss in numer of y-o-y recruits =

$$100\left(\frac{R_o - R_1}{R_o}\right) = 100\left(1 - \frac{R_1}{R_o}\right) \qquad (20)$$

where:

R_o = number of y-o-y recruits in Michigan waters in the absence of the Monroe power plant operation.

R_1 = number of y-o-y recruits in Michigan waters in the presence of the Monroe power plant operation.

In 1975:

$R_1 = 1.0\times10^7 - 2.9\times10^7$

$R_o = R_1 + R_2$

where:

R_2 = number of larvae that would have survived 25 days in the absence of entrainment.

Estimated number killed due to entrainment mortality in 1975 =

$= 1.5\times10^6 - 8.0\times10^6$

Percent (killed due to entrainment on 5th day of life) estimated

to have otherwise survived 25 days =

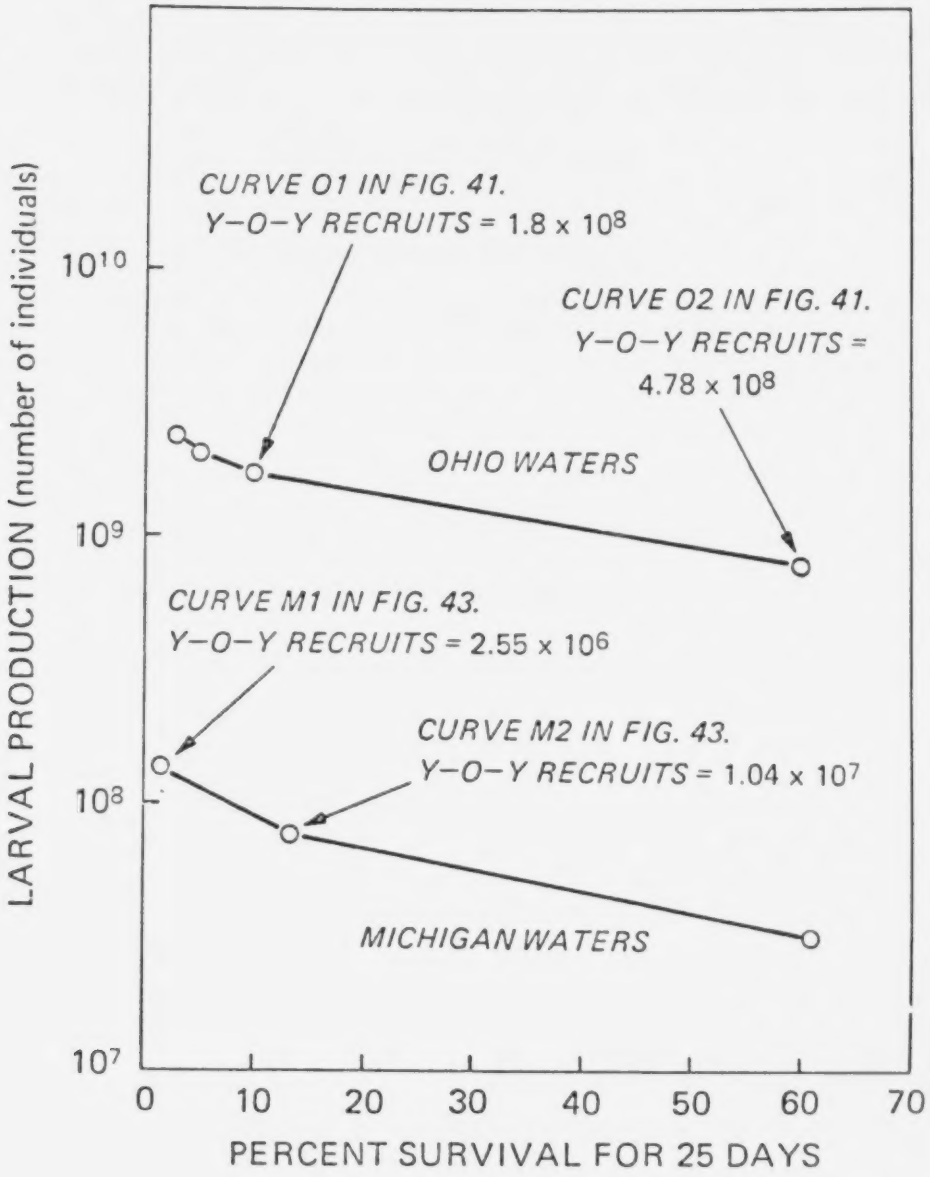

Figure 44. Plausible larval perch production - survival combinations in Western Basin (1976).

$= 5.5\% - 17.7\%$

$R_2 = 1.5\times10^6 \times .055 - 8.0\times10^6 \times .177 =$

$= 8.25\times10^4 - 1.42\times10^6$

Therefore, the estimated percent loss in number of y-o-y recruits is:

$$100\ (1 - \frac{R_1}{R_o}) = 100\ (1 - \frac{R_1}{R_1 + R_2}) = 100\ (1 - \frac{1.0\times10^7}{1.0\times10^7 + 8.25\times10^4})\ \text{to}$$

$$100\ (1 - \frac{2.9\times10^7}{2.9\times10^7 + 1.42\times10^6}) = 0.8\%\ \text{to}\ 4.7\%$$

In 1976:

$R_1 = 2.78\times10^6 - 8.4\times10^6$

$R_2 = 0.46\times10^6 \times .055 - 0.7\times10^6 \times .177 = 2.53\times10^4 - 1.24\times10^5$

Therefore, the estimated percent loss in number of y-o-y recruits is:

$$100\ (1 - \frac{R_1}{R_o}) = 100\ (1 - \frac{R_1}{R_1 + R_2}) = 100\ (1 - \frac{2.78\times10^6}{2.78\times10^6 + 2.53\times10^4})\ \text{to}$$

$$100\ (1 - \frac{8.4\times10^6}{8.4\times10^6 + 1.24\times10^5}) = 0.9\%\ \text{to}\ 1.5\%$$

SECTION 5

RESULTS AND DISCUSSIONS

Analysis of Losses to Standing Crop and the Fishery

The equations of balance for a population are composed of terms that parallel or reflect its life processes. Each term reflects assumptions about the dynamic behavior of a component process. When all processes are coupled through an equation of balance, temporal fluctuations in population are obtained which can then be studied by variation of process parameters.

In the following, an equation of balance which incorporates larval production, larval survival, young-of-year survival, natural mortality of subadults and adults, and fishing mortality is defined. Estimates of population parameters which permit a numerical analysis of the impact upon catch as a result of variations in any of these factors are provided.

Define the following variables and parameters:

$N(t)$ = adult population size (age class II and older fishes) in year t. [no. individuals].

$\dot{N}(t)$ = net annual instantaneous rate of change in adult population size in year t. $[\text{no.} \cdot \text{yr.}]^{-1}$.

f = mean annual instantaneous mortality rate from commercial and sport fishing. $[\text{yr.}^{-1}]$.

m = mean annual instantaneous mortality rate due to causes other than fishing, entrainment, and impingement. $[\text{yr.}^{-1}]$.

γ = mean annual rate of larvae production per individual in population size N. $[\text{no. larvae} \cdot \text{individual}^{-1} \text{yr.}^{-1}]$.

ε = fraction of larvae surviving environmental forces of mortality for first 25 days to reach young-of-year stage. $[\text{y.o.y.} \cdot \text{larvae}^{-1}]$.

s = annual fraction of young-of-year that survive until December 31 (of year in which they are produced) to be recruited into age class I (also referred to as yearling or sub-adult stage). [sub-adults · y.o.y.$^{-1}$].

e^{-m} = annual fraction of age class I sub-population that survives non-fishing causes of mortality to be recruited into adult population. [adults · sub-adults^{-1}].

K = habitat carrying capacity of adult population [no. adults].

T = maximum life length of adult fishes.

E_ℓ = annual loss of larval fishes due to power plant entrainment. [larvae · yr.$^{-1}$].

E_y = annual loss of young-of-year due to power plant entrainment. [y.o.y. · yr.$^{-1}$].

I_y = annual loss of young-of-year due to power plant impingement. [y.o.y. · yr.$^{-1}$].

I_{A1} = annual loss of age class I fishes due to power plant impingement. [sub-adults · yr.$^{-1}$].

I_N = annual loss of adults (age class II and older fishes) due to power plant impingement. [adults · yr.$^{-1}$].

L = annual larvae production rate. [larvae · yr.$^{-1}$].

I_A = annual loss of fishes in age class I and older due to power plant impingement. [adults · yr.$^{-1}$].

The verbal statement of population balance can be expressed as:

net annual instantaneous rate of change in population level

equals

annual rate of recruitement of sub-adults into age class II

minus

annual instantaneous rate of loss of stock due to fishing

minus

annual instantaneous rate of loss of stock due to non-fishing mortality

minus

annual rate of loss of adults which have survived the maximum age T.

In equation form:

$$\dot{N} = \alpha R - (m + f) \cdot N \tag{21}$$

or:

$$\dot{N} = \alpha R - (m + f) \cdot N - I_A \tag{21.1}$$

where:

$$\alpha = 1 - e^{-(m + f)\ (T - 1)}$$

Equation (21.1) defines a balance of the surviving population and equation (21) defines a balance on the segment of the population lost annually due to power plant entrainment and impingement mortality, and includes its lost reproductive potential.

The inferences made about impacts of entrainment and impingement mortality will depend upon how one represents or models R, the recruitment term in equations (21) and (21.1).

In the following, the hypothetical subpopulation of fishes absent as a result of entrainment and impingement mortality of larvae, juveniles, subadults, and adults will be analyzed using equation (21) as the basic expression for which the following recruitment model is considered:

Model 1:

$$R = E_\ell \cdot \varepsilon \cdot s \cdot e^{-m} + \quad \text{(entrained larvae component)}$$

$$+ E_y \cdot s \cdot e^{-m} + \quad \text{(entrained y.o.y. component)}$$

$$+ I_y \cdot s \cdot e^{-m} + \quad \text{(impinged y.o.y. component)}$$

$$+ I_{A1} \cdot e^{-m} + \quad \text{(impinged sub-adult component)}$$

$$+ I_N + \quad \text{(impinged adult component)}$$

$$+ \gamma \cdot N(t) \cdot \varepsilon \cdot s \cdot e^{-m} \quad \text{(reproductive potential component)}$$

Thus, if:

R = recruitment of individuals into age class II group; recruitment is expressed by the equation:

$$R = [I_N + e^{-m} (I_{A1} + s (I_y + E_y + \varepsilon (E_\ell + \gamma \cdot N(t))))] \tag{22}$$

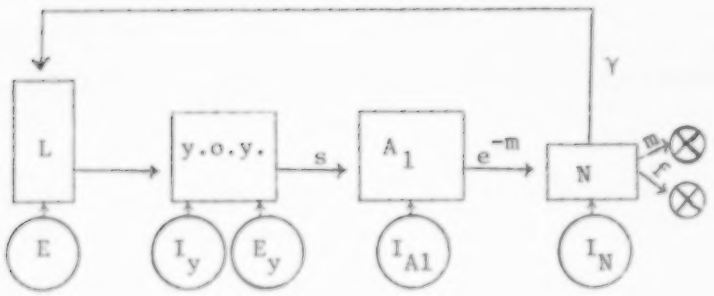

Diagram B: Conceptual Model of Hypothetical Population

A numerical analysis of the size of the unrealized subpopulation of fishes follows by substituting equation (22) into (21), solving for N, and calculating a steady state population size together with fishing harvest for different feasible combinations of population parameters. Diagram B is a flow chart of the resulting materials balance.

A second model of recruitment was also considered in which equation (21) represents the net balance for the surviving population rather than the hypothetical subpopulation of fishes not present due to entrainment and impingement mortality. The analysis of the second model is limited to a small number of combinations of population parameters and is included to indicate the potential compensatory effects within the perch population in Lake Erie.

Model 2:

$R = s \cdot \varepsilon \cdot \gamma \cdot N(t) \left(1 - \frac{N(t)}{K}\right)$ reproduction and recruitment prior to adjustment for entrainment and impingement.

$- s \cdot \varepsilon \cdot E_\ell$ entrained larvae component.

$- s (E_y + I_y)$ entrained and impinged young-of-year component.

Thus, recruitment is expressed by the equation:

$$R = s[\varepsilon \cdot (\gamma \cdot N(t) \left(1 - \frac{N(t)}{K}\right) - E_\ell) - (E_y + I_y)] \qquad (23)$$

Model 2 incorporates a parameter, K, representing the habitat carrying capacity for the population. The carrying capacity may change slowly over time as water quality and interspecific factors of competition change. The carrying capacity represents the upper limit of the attainable size of the population. If the population is in a state of dynamic equilibrium, it will always be at a level below the carrying capacity. As the carrying capacity changes, the equilibrium level of the population will adjust itself to a new

value, again below the new value of K. The amount by which the equilibrium value of the population lies below the carrying capacity depends upon the other population parameters, as well as losses to the population represented by the terms E_ℓ, I_y, E_y, and I_A. The dynamics of recruitment in equation (23) are such that the population increase follows an S-shaped curve, approaching its equilibrium value. The rate of population increase slows down as population density increases, due to a reduced rate of recruitment of larvae into the juvenile stage. Equation (23) is, no doubt, the simplest way to introduce compensation for population density into the dynamics of recruitment. It should be noted that in equation (23) the product $\varepsilon \cdot \gamma$ (number of larvae per individual surviving to enter young-of-year stage) is multiplied by the compensation term $(1 - \frac{N}{K})$, rather than ε or γ alone.[1] Thus, the expression $\varepsilon \cdot \gamma \cdot (1 - \frac{N}{K})$ is used to approximate the actual, but unknown function $\varepsilon(N) \cdot \gamma(N)$ describing larval survival at 25 days following hatching of eggs. Furthermore, there is no attempt in equation (23) to model changes in entrainment and impingement mortality brought about by fluctuations in larval production from one year to the next. The terms E_ℓ, E_y, and I_y are constant throughout but can be varied from one calculation of equilibrium population to the next.

The effect of entrainment or impingement mortality is analyzed by modeling the whole population rather than the subpopulation of entrained and impinged fishes as in the earlier case. Although the quadratic term in equation (23) creates the S-shaped curve of population change as it approaches equilibrium, a more important characteristic of the equation for present purposes is the manner in which the equilibrium value of the population is limited by the population parameters and by entrainment and impingement losses.

Solutions to Equations

Substituting equation (22) into (21) and solving for N, one obtains:

$$N(t) = N(0)e^{-\beta t} + \alpha \int_0^t R(x)e^{-\beta(t-x)}\,dx \qquad (24)$$

where:

$$\beta = m + f - \alpha \cdot s \cdot e \cdot \gamma \cdot e^{-m},^2$$

$$R(x) = I_N(x) + e^{-m}[I_{A1}(x) + s(I_y(x) + E_y(x) + \varepsilon \cdot E_\ell(x))]$$

and:

$N(0)$ = initial population size.

---

[1] Other quantitative expressions for representing compensatory effects of high larval mortality are currently in use (20). Research and modeling of compensatory processes is active.

[2] Dimensional analysis verifies that $\alpha \cdot \varepsilon \cdot s \cdot \gamma \cdot e^{-m}$ and $(m + f)$ are comparable quantities.

$I_N(x)$, $I_{A1}(x)$, $I_y(x)$, $E_\ell(x)$ and $E_y(x)$ may be constants or functions of the time parameter x. As time t increases, the contribution of the initial population level N(0) diminishes exponentially. The integral term is an exponentially weighted moving average of the contributions of successive recruitments R(x) in year x in which more distant additions R(x) contribute an exponentially decreasing proportion to the total population. If one is interested in a steady state condition, as t approaches infinity, N(t) approaches, under appropriate conditions on R(x),

$$N = \lim_{t \to \infty} \alpha \int_0^t R(x) e^{-\beta(t-x)} dx$$

This is the steady state value of the size of the hypothetical subpopulation lost due to entrainment and impingement. In the following, all impingement and entrainment functions are assumed to be constants and therefore independent of time. Therefore, $R(x) = \bar{R}$ = constant.
For this case :

$$N(t) = [N(0) - \frac{\alpha}{\beta} \bar{R}] e^{-\beta t} + \frac{\alpha}{\beta} \cdot \bar{R} \qquad (25)$$

where R is defined above, and:

$$\alpha = 1 - e^{-(m + f) \cdot 6}$$

where:

$$T - 1 = 6.$$

Equation (25) shows no steady state unless $m + f > \alpha \cdot \varepsilon \cdot s \cdot \gamma \cdot e^{-m}$, i.e., $\beta > 0$, which must be true in the environment on the average or the population would explode. When $\beta > 0$, the steady state population is:

$$\frac{\alpha}{\beta} \cdot R$$

Model 2 of recruitment is exercised by substituting equation (23) into equation (21.1) and solving the differential equation:

$$\dot{N} = \alpha R - (m + f) \cdot N - I_A =$$

$$= \alpha \cdot s \cdot \varepsilon \cdot \gamma \cdot N \cdot (1 - \frac{N}{K}) - \alpha \cdot s \cdot (E_y + I_y + \varepsilon \cdot E_\ell) - I_A -$$

$$- (m + f) \cdot N$$

where:

$$\alpha = 1 - e^{-(m + f) \cdot 7}$$

Collecting coefficients of N^0, N^1, N^2:

$$\dot{N} = aN^2 + bN + c \qquad (26)$$

where:

$$a = -\frac{\alpha \cdot s \cdot \varepsilon \cdot \gamma}{K}, \quad b = \alpha \cdot s \cdot \varepsilon \cdot \gamma - (m + f)$$

and:

$$c = -(\alpha \cdot s \cdot (E_y + I_y + \varepsilon \cdot E_\ell) + I_A).$$

Solving:

$$N(t) = A + \frac{B-A}{1 + \left(\frac{B - N_o}{N_o - A}\right) e^{a(B-A)t}} \tag{27}$$

where:

N_o = initial population size,

Thus:

$$A = \frac{c}{a} \cdot B^{-1},$$

and:

$$B = \frac{1}{2}\left[-\frac{b}{a} + \sqrt{\left(\frac{b}{a}\right)^2 - 4\frac{c}{a}}\right]$$

The equilibrium value of the population is B, assuming $B < A$. The value of the coefficient a indicates rate of recovery of the population from a disturbance. A negative value of b indicates extinction of the population. Such a condition occurs if the combined natural and fishing mortality rate exceeds the reproductive potential of the population. The population can also decline until it reaches zero if the loss terms E_ℓ, E_y, I_y, and I_A are sufficiently large.

Estimates of Entrainment and Impingement Mortality

Cole (4) estimated the following 95 percent confidence intervals of the numbers of yellow perch larvae potentially entrained at the Monroe power plant in 1973, 1974, and 1975:

	Estimated Number Entrained (millions)	Year
(1)	$0 \leq 2.2 \leq 5.1$	1973
(2)	$59.6 \leq 83.1 \leq 111.5$	1974
(3)	$13.7 \leq 29.3 \leq 44.9$	1975

Using sampled concentrations of larvae obtained by Cole in the river channel and in the upper discharge channel and volumes of cooling water published in (7), the author obtained the following three estimates of numbers of perch larvae entrained in the power plant cooling waters in 1975.

Estimated Number Entrained (millions)	Location of Measurement	Year
(4) 2.72 - 14.26	river channel near mouth	1975
(5) 2.39 - 20.85	upper discharge	1975
(6) 19.4	lake waters near river mouth	1975
(7) 0.94		1976

The Detroit Edison Company reports an estimated 5,029,000 perch larvae entrained in 1975 (7) which includes prejuveniles or young-of-year fishes. Since prejuveniles were not counted separately, it is assumed that combined young-of-year mortality due to entrainment and impingement is:

$$I_y + E_y = 100,000$$

This assumption is probably conservative in view of the estimated total number entrained as reported by Detroit Edison. Based upon Cole's estimate of 20 percent of perch larvae either dead or dying prior to entrainment and using an estimate of 70 percent mortality of live perch larvae entrained, the estimates of numbers of larvae entrained can be reduced to E_ℓ, an estimate of live larvae killed due to entrainment:

TABLE 22. ESTIMATES OF ENTRAINMENT CAUSED BY LARVAL MORTALITY.

	E_ℓ (millions killed)	
(1)	0-2.856	(1973)
(2)	33.38-62.44	(1974)
(3)	7.672-25.14	(1975)
(4)	1.526-7.986	(1975)
(5)	1.338-11.68	(1975)
(6)	10.86	(1975)
(D.E.)	2.816	(1975)
(7)	0.53	(1976)
(D.E.)	0.36	(1976)

Estimated Impingement Mortality

Detroit Edison published the following estimates of numbers of fishes killed due to impingement (7):

TABLE 23. ESTIMATED IMPINGEMENT MORTALITY.

$(I_y + I_{A1} + I_A)$	Year
165,365 (excluding Jan., Feb., Mar.)	1972
215,032	1973
152,857	1974
171,641 (excluding April and May)	1975

It is impossible to estimate the individual terms I_y, I_{A1}, I_A from the data shown above. Therefore, for computational purposes, the quantities I_A and I_{A1} (adult and subadult mortality, respectively) are each permitted to assume the values 0, 50,000, and 100,000 independently, so that the sum $I_A + I_{A1}$ ranges from 0 to 200,000. When the sum $I_A + I_{A1}$ is combined with the earlier assumption of $I_y + E_y = 100,000$, it is clear that total impingement mortality will range over the values shown above.

Estimates of Population Parameters

ε: fraction of larvae surviving natural environmental forces of mortality to reach young-of-year stage.

The methodology underlying the estimate of ε, the fractional rate of survival of larvae from natural environmental forces of mortality to reach young-of-year stage (25 days after date of hatching), is based upon the above analysis of larval production in U.S. waters of the western basin of Lake Erie and resulted in an estimate of ε in the range[1]:.

$$0.02 \leq \varepsilon \leq 0.10$$

s: fraction of young-of-year that survives to be recruited into age class I.

Data (7) indicating abundance of young-of-year and yearlings in 4 successive years (1972, 1973, 1974, 1975) yield estimates of annual young-of-year survival fractions of 0.12, 0.19, and 0.33, respectively, using the ratio[2]:

$$\text{y.o.y. survival fraction} = \frac{\text{yearlings C.P.E. in year (t)}}{\text{y.o.y. C.P.E. in year (t-1)}}$$

[1]It is shown in Appendix H that the percentage of entrained larvae that would have survived for 25 days had they not been entrained increases with age at entrainment and may reach 25%.

[2]A constant coefficient of catchability is assumed.

The monthy instantaneous mortality rates, α, corresponding to annual survival fractions of 0.12 and 0.33 are, respectively,

$$\alpha = -\frac{1}{12} \ln .12 = 0.177$$

and

$$\alpha = -\frac{1}{12} \ln .33 = 0.092$$

In turn, the monthly instantaneous rates 0.177 and 0.092 yield six months survival fractions which are, respectively,

$$s = e^{-.177(6)} = 0.346$$

and

$$s = e^{-.092(6)} = 0.575.$$

Thus, the fraction of young-of-year that survives (approximately 6 months) to be recruited into the yearling stage or age class I is[1]:

$$0.34 \leq s \leq 0.58.$$

f: annual instantaneous fishing mortality rate from commercial and sport fishing.

Based upon an estimate by Jensen (School of Natural Resources, University of Michigan) that 20 to 40 percent of all perch vulnerable to fishing gear will be harvested annually by commercial or sport fisherman, the instantaneous annual fishing mortality rate is estimated to be between 0.22 and 0.51. However, it is reported in (22) that during the period 1968 to 1978, total annual mortality may have risen to 70 percent. If the annual natural mortality fraction holds at 25 percent, the implication follows that the instantaneous annual fishing mortality rate may have increased to 0.95. Therefore, it is estimated as:

$$.22 \leq f \leq 0.95$$

m: annual instantaneous natural mortality rate of yellow perch.

The Great Lakes Fishery Laboratory, U.S. Fish and Wildlife Service[2] reports an estimated natural mortality rate in the range:

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[1] It is reported in (22) that research on post-larval yellow perch mortality indicates a six month survival of approximately 0.49, a value well within the interval (.34, .58).

[2] Reference (22) and W.L. Hartman (Great Lakes Fishery Laboratory, U.S. Fish and Wildlife Service).

$$0.22 \leq m \leq 0.29$$

Other data relating to natural mortality of yellow perch are reported in (14) and (15).

γ: mean annual rate of larvae production per individual in the subpopulation.

In model 1, the subpopulation for which γ is estimated includes all fishes in age class II and older. In model 2, the subpopulation estimated includes all fishes in age class I and older. By definition:

γ = hatching success · number eggs deposited per sexually mature female spawner

· number sexually mature female spawners per individual in subpopulation.

It is estimated by Scholl (Division of Wildlife, Ohio Department of Natural Resources) that hatching success for yellow perch in Lake Erie ranges between 25 and 50 percent:

$$.25 \leq \text{hatching success} \leq .50$$

The number of eggs deposited per sexually mature female in Lake Ontario is reported by Scott and Crossman (23) to vary between 2,000 and 90,000 with an estimated average of 23,000. For the present calculation pertaining to Lake Erie, it is therefore assumed that the number of eggs deposited per sexually mature female ranges from 10,000 to 30,000.

The number of sexually mature female spawners per individual in the subpopulation is variable and depends upon the mean number of times that a female spawns during her lifetime, the number of age classes included in the subpopulation, and the mean mortality rates from both natural causes and fishing. Given equilibrium population conditions and assuming that the subpopulation of interest consists of all fishes in age class II and older, the number of sexually mature female spawners per individual in the subpopulation can be calculated.

To do so, N_2 is used to denote the number of individuals entering age class II under equilibrium conditions. Based upon Ohio Division of Wildlife (12) information on sexual maturation of yellow perch of different ages, it can be assumed that no age class II females are sexually mature and that all females in age classes III and older are sexually mature. Assuming that a fraction ρ_f of any age class are females, that a sexually mature female spawns in any given year with probability ρ_s, that fishing mortality commences with age class III individuals, and that the equilibrium number of age class I individuals is N_1, we may calculate the fraction of sexually mature female spawners in the subpopulation of age class II and older fishes from the following:

Number of Individuals	Number of Female Spawners	Age Class
N_1e^{-m}	0	II
$N_1e^{-(2m + f)}$	$\rho_s\rho_f N_1e^{-(2m + f)}$	III
$N_1e^{-(3m + 2f)}$	$\rho_s\rho_f N_1e^{-(3m + 2f)}$	IV
$N_1e^{-(4m + 3f)}$	$\rho_s\rho_f N_1e^{-(4m + 3f)}$	V
$N_1e^{-(5m + 4f)}$	$\rho_s\rho_f N_1e^{-(5m + 4f)}$	VI
$N_1e^{-(6m + 5f)}$	$\rho_s\rho_f N_1e^{-(6m + 5f)}$	VII

The fraction of sexually mature female spawners in the subpopulation may be defined as:

number of sexually mature female spawners per individual in subpopulation (age class II - VII) =

$$\frac{\rho_s\rho_f[e^{-(m + f)} + \cdots + e^{-5(m + f)}]}{[1 + e^{-(m + f)} + \cdots + e^{-5(m + f)}]}$$

It may then be deduced that the fraction of the subpopulation consisting of sexually mature females drops as the fishing and natural mortality rates increase. For example, if $f = m = 0.22$, the above fraction is $\rho_s \cdot \rho_f \times .60$ but if $f = m = .52$, the fraction drops to $\rho_s \cdot \rho_f \times .35$. By assuming (a) that fifty percent[1] of each age class consists of females and (b) that the probability of spawning by any given sexually mature female is .8 to 1.0, and by using the ranges of m and f estimated above, we calculate the number of sexually mature females per individual in the subpopulation consisting of all fishes in age class II and older to vary between 0.153 and 0.239:

$$0.15 \leq \begin{matrix}\text{number of sexually mature female} \\ \text{spawners per individual in} \\ \text{subpopulation (age class II-VII)}\end{matrix} \leq 0.24$$

Therefore, the parameter γ is estimated to lie in the range:

$$(0.25)\ (10{,}000)\ (0.15) = 375 \leq \gamma \leq (0.50)\ (30{,}000)\ (0.24) = 3600.$$

The estimated mean number of recruits into age class II per year per individual in the subpopulation under equilibrium conditions, $\gamma \cdot \varepsilon \cdot s \cdot e^{-m}$, follows:

$$(375)\ (0.02)\ (0.22)\ (0.75) \leq \gamma \cdot \varepsilon \cdot s \cdot e^{-m} \leq (3600)\ (.10)\ (.58)\ (.80)$$

namely:

$$1.23 \leq \gamma \cdot \varepsilon \cdot s \cdot e^{-m} \leq 167.5$$

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[1]Sex ratios among yellow perch do not remain constant at 50:50 across age classes. This assumption may be an overestimate of the actual percentages of females in the older age classes. Recent data on sex ratios of yellow perch in Lake Erie are unavailable.

In the long run, the mean rate of addition of recruits to the subpopulation must not exceed the total mean rate of removals.

Impacts of Entrainment and Impingement Mortality

The reduction in yield from age class II and older yellow perch in Michigan-Ohio waters of the western basin has been estimated by calculating an equilibrium size of a subpopulation created with the use of the continued annual losses of larvae, juveniles, and adults incurred from entrainment and impingement mortality as input. When the reproductive potential of this population is taken into consideration, when it is assumed that such a population is subject to the same biological processes and environmental pressures as the surviving population, one can use the parameter estimates given above together with equation (25) to estimate the size N of this hypothetical population. One then estimates the loss in annual yield to sport and commercial fishermen by multiplying the estimated equilibrium value of N by the annual fishing mortality fraction $1-e^{-f}$.

A computer program was written and sizes, N, of an equilibrium population and $(1-e^{-f})$ N, loss in yield, were calculated:

$$N = \alpha \cdot \beta^{-1} \cdot [I_N + e^{-m}(s(E_\ell \cdot \varepsilon' + 100{,}000) + I_{A1})]$$

where:

$$\alpha = 1 - e^{-(m + f) \cdot 6},$$

ε' = fraction of entrained larvae estimated to have survived to reach young-of-year stage.

and:

$$\beta = m + f - \alpha \cdot s \cdot \varepsilon' \cdot \gamma \cdot e^{-m} \quad (\beta > 0 \text{ is required for equilibrium})$$

TABLE 24. VALUES OF POPULATION PARAMETERS AND ENTRAINMENT AND IMPINGEMENT MORTALITIES USED IN CALCULATION OF POTENTIAL IMPACT ON POPULATION SIZE.

ε':	.08, .13[1]
s:	.42, .50
f:	.52, .95
m:	.29
γ:	15
E_ℓ:	2×10^6, 10×10^6, 20×10^6, 40×10^6
I_N:	0, 50,000, 100,000
I_{A1}:	0, 50,000, 100,000

Calculations of potential losses in yield for combinations shown in Table 24 are given in Tables 25-32. The parmaters m and γ are held constant in each case.

[1]Calculations given in Appendix H provide a bases for setting ε' = .08 and .13.

The value $\gamma = 15$ is held constant and provides a modest reproductive potential of the population of fishes lost due to entrainment and impingement. The values of ε', s, and f form eight combinations each of which corresponds to one of the tables. In each case a constant loss of young-of-year due to entrainment and impingement ($I_y + E_y$) is set equal to 100,000 individuals. Three levels of losses of yearlings and adults ($I_{A1} + I_N$) are considered (0, 1×10^5, 2×10^5) and in each case the loss is split equally ($I_N = I_{A1}$) between yearlings and older fishes. All losses in yield are expressed in pounds of fish, assuming 3.5 fish per pound. The entries in Tables 25-32 are calculated from equations (28) to (35), respectively.[1]

The coefficients of I_N, I_{A1}, and E_ℓ in equations (28) to (35) convert annual losses of fishes in each of the three stages (due to entrainment and impingement) into reductions in yield to the fisheries.

Thus, for example, an annual loss of 1 million larvae converts into a potential annual loss of $1 \times 10^6 \times .007 = 7000$ pounds to the fisheries.

TABLE 25. ESTIMATED POTENTIAL LOSS IN YIELD (POUNDS).

$I_{A1} + I_N$	$\varepsilon' = .08$; s = .42; f = .52			
200,000	68262	124262	194262	334262
100,000	45262	101262	171262	311262
0	22262	78262	148262	288262
	2.0	10.0	20.0	40.0

E_ℓ: (millions)

Loss in yield: $0.263\ I_N + 0.197\ I_{A1} + 0.007\ E_\ell + 8262$ (28)

TABLE 26. ESTIMATED POTENTIAL LOSS IN YIELD (POUNDS).

$I_{A1} + I_N$	$\varepsilon' = .08$; s = .42; f = .95			
200,000	52116	93259	144687	247544
100,000	34387	75530	126959	229816
0	16658	57801	109230	212087
	2.0	10.0	20.0	40.0

E_ℓ: (millions)

Loss in yield = $0.203\ I_N + 0.152\ I_{A1} + .005\ E_\ell + 6373$ (29)

[1]Each of the equations (28) to (35) is a special case of the equation for N given above, namely the size of the hypothetical population of equivalent adults lost due to entrainment and impingement.

TABLE 27. ESTIMATED POTENTIAL LOSS IN YIELD (POUNDS).

$I_{A1} + I_N$	$\varepsilon' = .08$; $s = .50$; $f = .52$			
200,000	84643	156643	246643	426643
100,000	57193	129193	219193	399193
0	29743	101743	191743	371743
	2.0	10.0	20.0	40.0

E_ℓ: (millions)

Loss in yield: $0.314\ I_N + 0.235\ I_{A1} + 0.009\ E_\ell + 11743$ (30)

TABLE 28. ESTIMATED POTENTIAL LOSS IN YIELD (POUNDS).

$I_{A1} + I_N$	$\varepsilon' = .08$; $s = .50$; $f = .95$			
200,000	60865	116865	186865	326865
100,000	41565	97565	167565	307565
0	22265	78265	148265	288265
	2.0	10.0	20.0	40.0

E_ℓ: (millions)

Loss in yield = $0.221\ I_N + 0.165\ I_{A1} + 0.007\ E_\ell + 8265$ (31)

TABLE 29. ESTIMATED POTENTIAL LOSS IN YIELD (POUNDS).

$I_{A1} + I_N$	$\varepsilon' = .13$; $s = .42$; $f = .52$			
200,000	163144	347144	577144	1037144
100,000	113494	297494	527494	987494
0	63844	247844	477844	937844
	2.0	10.0	20.0	40.0

E_ℓ: (millions)

Loss in yield = $0.568\ I_N + 0.425\ I_{A1} + 0.023\ E_\ell + 17844$ (32)

TABLE 30. ESTIMATED POTENTIAL LOSS IN YIELD (POUNDS).

$I_{A1} + I_N$	$\varepsilon' = .13$; $s = .42$; $f = .95$			
200,000	79565	167565	277565	497565
100,000	55165	143165	253165	473165
0	30765	118765	228765	448765
	2.0	10.0	20.0	40.0

E_ℓ: (millions)

$$\text{Loss in yield} = 0.279\ I_N + 0.209\ I_{A1} + 0.011\ E_\ell + 8765 \qquad (33)$$

TABLE 31. ESTIMATED POTENTIAL LOSS IN YIELD (POUNDS).

$I_{A1} + I_N$	$\varepsilon' = .13$; $s = .50$; $f = .52$			
200,000	408068	920068	1560068	2840068
100,000	292718	804718	1444718	2724718
0	177368	689368	1329368	2609368
	2.0	10.0	20.0	40.0

E_ℓ: (millions)

$$\text{Loss in yield} = 1.32\ I_N + 0.987\ I_{A1} + 0.064\ E_\ell + 49368 \qquad (34)$$

TABLE 32. ESTIMATED POTENTIAL LOSS IN YIELD (POUNDS).

$I_{A1} + I_N$	$\varepsilon' = .13$; $s = .50$; $f = .95$			
200,000	106590	242590	412590	752590
100,000	76690	212690	382690	722690
0	46790	182790	352790	692790
	2.0	10.0	20.0	40.0

E_ℓ: (millions)

$$\text{Loss in yield} = 0.342\ I_N + 0.256\ I_{A1} + 0.017\ E_\ell + 12790 \qquad (35)$$

Since the annual loss in yield is never constant even under equilibrium conditions, the estimates given in Tables 25-32 are reduced to estimates of a single weighted mean annual loss for each of the two values of ε' that were selected. In order to carry out this reduction, a probability distribution must be assigned to the values appearing in Tables 25-28 and Tables 29-32. Such an assignment should give recognition to the estimates contained in Tables 22 and 23. It should also reflect estimates of recent

fishing pressure and young-of-year survival which are assumed to be independent of each other. Based upon an interpretation of Tables 22 and 23, the following distribution of weights is assigned to each table entry for Tables 25-28. The distribution is applied again to Tables 29-32, so that two weighted mean estimates of potential annual loss in yield are obtained.

TABLE 33. PROBABILITY WEIGHTS ASSIGNED TO EACH TABLE ENTRY.

	$I_{A1} + I_N$	Wt: .25 each (s,f) combination			
.088	200,000	.016	.054	.016	.002
.162	100,000	.029	.101	.029	.003
0	0	0	0	0	0
	E_ℓ:	2.0	10.0	20.0	40.0
	Wt:	.045	.155	.045	.005

The entries in the main body of the tables are obtained by multiplying each combination (E_ℓ, $I_{A1} + I_N$, s, f) by the respective weights contained in the marginal distributions on E_ℓ, $I_{A1} + I_N$, and (s, f). The marginal distributions assigned to E_ℓ, $I_{A1} + I_N$, are given in Tables 34 and 35.

TABLE 34. MARGINAL PROBABILITY DISTRIBUTION ASSIGNED to $I_{A1} + I_N$.

$I_{A1} + I_N$	Wt. Assigned
200,000	.35
100,000	.65
0	0

TABLE 35. MARGINAL PROBABILITY DISTRIBUTION ASSIGNED to E_ℓ.

E_ℓ	Wt. Assigned
40×10^6	.02
20×10^6	.18
10×10^6	.62
2×10^6	.18

The overall weighted mean estimates of the potential annual loss in fishery yield are approximately 110,000 pounds and 406,000 pounds for the cases $\varepsilon' = .08$ and $\varepsilon' = .13$, respectively. Therefore, the potential mean

annual loss is estimated to be 110,000-406,000 pounds. An annual loss yield of 100,000 pounds is not great when compared to a realized annual yield of 5 to 6 million pounds and would be extremely difficult if not impossible to detect by statistical methods applied to standing crops or harvests, except locally. However, it has been demonstrated that losses due to impingement and entrainment must occur and an average actual reduction in total harvest over a period of years is expected. If fishing pressure is increased in an attempt to maintain catch, as has occurred in recent years, the fisheries are perturbed in the direction of overexploitation and an eventual drop in harvest due directly to power plant impacts cannot be rectified by biological compensation. The estimates given above apply only to the Monroe power plant. With additional plants with once-through cooling being situated in the western basin, it is easily seen that their combined pressure on the fisheries will be substantial, and, when this is coupled with excessive harvests, it could tip the yellow perch fisheries into an irreversible decline. It is clear from examination of catch effort and stock assessment data collected for the past twenty years that the combined yellow perch harvest has been declining for the past seven to nine years, and the population is presently in a depressed condition. The major reasons for this condition are over exploitation and poor recruitment, but the fact remains that entrainment and impingement mortality from power plant cooling waters is exercising an impact upon the fisheries. If fishing pressure by Canada and the U.S. were relaxed by 10 percent per year, the immediate effect would be a reduction of the harvest, but over a period of years the population would recover a substantial portion of its reproductive base and yields would increase above present levels. Under such conditions, entrainment and impingement mortality will actually increase in absolute terms (numbers entrained and impinged), rather than decrease. The differential impact of entrainment and impingement mortality, however, would be lessened, due to the presence of a larger reproductive base. The impact of a given level of entrainment and impingement mortality upon the yellow perch population is most severe when the population is in a depressed condition, as is the present situation. This analysis, based upon equation (25), is valid only so long as there is sufficient reproductive stock to maintain an equilibrium population in the presence of the array of natural mortality, fishing mortality, and entrainment and impingement mortality. As losses increase, a point is reached at which where an equilibrium population is not possible and the fishery collapses.

Effects of Compensation

Effects of compensation, if any, by the surviving population may already be accounted for in the losses estimated above. If the compensation fraction is denoted by δ, the actual loss in yield is estimated as:

$$\text{actual loss in yield} = \text{potential loss} \times (1-\delta) = (1-e^{-f})\, N\, (1-\delta) \quad (36)$$

where:

$$0 \leq \delta \leq 1$$

and:

$$N = \frac{\alpha}{\beta}\left[I_N + e^{-m}\left(I_{A1} + s(E_\ell \cdot \varepsilon' + 100{,}000)\right)\right]$$

The present level of compensation by yellow perch in Lake Erie is unknown in numerical terms but may well be at its maximum, if it occurs at all.[1] In order to obtain some indication of the degree to which compensatory mechanisms might mitigate the combined impacts of overexploitation, entrainment and impingement mortality, and adverse environmental conditions resulting in poor survival of larvae and young-of-year, a simplistic model of recruitment (Model 2) defined by equation (23) was solved (equation 27)) and a set of calculations were made assuming equilibrium conditions in the population (Table 36). Note from equation (26) that the compensatory term affects the rate of recruitment into the age class I subpopulation which is:

$$\alpha \cdot s \cdot \varepsilon \cdot \gamma \cdot N \cdot \left(1 - \frac{N}{K}\right)$$

the rate of recruitment per individual into the age class I and older population is:

$$\alpha \cdot s \cdot \varepsilon \cdot \gamma \cdot \left(1 - \frac{N}{K}\right)$$

Since compensation undoubtedly occurs separately through the terms γ and ε, implying that both γ and ε are functions of N, the above expression may be considered to be a first order representation of some actual but unknown compensatory mechanism operative in the population.

Table 36 indicates that if reproductive potential is high (γ = 300 - 1500), compensation can eliminate the effects of entrainment and impingement losses. However, when reproductive potential is low (γ = 75), compensation is much less effective. As reproductive potential decreases even further (γ = 50), compensation cannot prevent a total collapse in the population under conditions of high fishing pressure and moderate losses due to entrainment and impingement mortality. This analysis of compensatory effects suggests that under the present conditions of a depressed yellow perch fishery, the effect of any additional compensatory reserve operative in the population is slight if it exists at all.

Appendix G indicates the statistical variation in the equilibrium population level as ε and γ fluctuate from year to year. Although historical data on year class strength suggests that annual larval survivals are correlated so that annual fluctuations may be less than the value calculated, natural environmental factors creating large variations in population size are sufficient to mask smaller systematic annual losses imposed by man.

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[1]Evidence contradicting the presence of compensation within yellow perch populations is cited in (Smith 1977, JFRBC 34(10): 1774-1783.

TABLE 36. EQUILIBRIUM HARVEST UNDER DIFFERENT CONDITIONS OF FISHING PRESSURE, REPRODUCTIVE POTENTIAL, AND ENTRAINMENT AND IMPINGEMENT. LOSSES: $K = 5 \times 10^7$; $\varepsilon = .08$; $s = .26$; $m = .37$.

γ	I_A	$E_y + I_y$	E_ℓ	f	Annual Harvest (lbs.)
1500	0	0	0	.37	4,312,660
1500	1×10^5	1×10^5	1×10^7	.37	4,311,976
1500	1×10^5	1×10^5	1×10^7	.52	5,625,718
1500	1×10^5	1×10^5	2×10^7	.37	4,311,082
1500	1×10^5	1×10^5	2×10^7	.52	5,624,922
300	1×10^5	1×10^5	1×10^7	.37	3,885,591
300	1×10^5	1×10^5	1×10^7	.52	4,957,475
300	1×10^5	1×10^5	2×10^7	.37	3,882,235
300	1×10^5	1×10^5	2×10^7	.52	4,952,952
300	0	0	0	.37	3,890,979
75	0	0	0	.37	2,309,674
75	1×10^5	1×10^5	1×10^7	.37	2,272,832
75	1×10^5	1×10^5	1×10^7	.52	2,431,838
75	1×10^5	1×10^5	2×10^7	.37	2,249,305
75	1×10^5	1×10^5	2×10^7	.52	2,393,347
50	0	0	0	.37	1,255,471
50	1×10^5	1×10^5	1×10^7	.37	1,145,857
50	1×10^5	1×10^5	1×10^7	.52	0
50	1×10^5	1×10^5	2×10^7	.37	1,064,052
50	1×10^5	1×10^5	2×10^7	.52	0

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APPENDIX A

STATISTICAL TESTS OF SIGNIFICANCE FOR DIFFERENCE IN CONCENTRATIONS OF LARVAL YELLOW PERCH IN THE WESTERN BASIN OF LAKE ERIE IN MAY AND JUNE 1975
Data Sources: Ref. (2,4,5)

Introduction and Summary

Accurate estimates of abundance of larval yellow perch depend upon the attainment of unbiased estimates of mean concentrations in the water column obtained at frequent intervals throughout a six week period beginning approximately May 1, of a given year. Many factors influence the reliability and accuracy of such estimates including sampling frequency, time, location, and equipment. Field samples obtained by teams from the Michigan State University Institute of Water Research and Ohio State University Center for Lake Erie Area Research suggest that larval yellow perch are highly non-uniformly distributed in the water column and moreover, this non-uniform distribution varies diurnally. Nine statistical tests of significance follow below which deal with differences in larval concentrations observed to exist between the surface and bottom of the water column during hours of both daylight and darkness. These tests indicate that substantial day - night differences exist at both the top and bottom of the water column in Michigan waters of the western basin. One important exception occurs in the vicinity of the mouth of the Maumee River where no significant differences in concentration was detected between surface and bottom during daylight hours. Significant surface - bottom differences in larval concentrations exist in most Ohio waters in which yellow perch spawning occurs.

The above results indicate that it is necessary to sample the water column at both surface and bottom during hours of darkness in order to accurately estimate larval concentrations and abundance. It is important to note in Table 2 that bottom sled tows yielded substantially higher concentrations of larval perch than nets towed near bottom in the same general vicinity.

Tests of Hypotheses Concerning Significance of Observed Differences in Mean Concentrations

Null Hypothesis 1:

Mean daytime concentrations at surface and near bottom in Michigan waters are equal.

Alternative: Concentration near bottom is greater.

Data:

$\bar{x}_S$	= 1.97*	$\bar{x}_B$	= 2.43
s_S	= 3.99	s_B	= 4.28
n	= 48	n	= 48

Test Statistic Value:

$$t = \frac{2.43 - 1.97}{\left[\frac{(4.28)^2}{48} + \frac{(3.99)^2}{48}\right]^{\frac{1}{2}}} = \frac{0.46}{0.84} = 0.54$$

$$\text{d.f.} = \frac{\left[\frac{(4.28)^2}{48} + \frac{(3.99)^2}{48}\right]}{\frac{\left[\frac{(4.28)^2}{48}\right]^2}{49} + \frac{\left[\frac{(3.99)^2}{48}\right]^2}{49}} - 2 =$$

$$= \frac{0.5088}{0.0052} - 2 = 95$$

Result: Since $t_{.70} = 0.527$ for 95 d.f., the null hypothesis is supported by the data, i.e., there is more than a 30 percent chance that random effects alone could produce a value of t = 0.54 under the null hypothesis.

Null Hypothesis 2:

Mean daytime concentrations at surface and on bottom in Michigan waters are equal.

Alternative: Concentration on bottom is greater.

Data:

$\bar{x}_S$	= 1.97	$\bar{x}_B$	= 5.83
s_S	= 3.99	s_B	= 4.52
n	= 48	n	= 15
			(bottom sled tow)

---

* $\bar{x}$. = sample mean.
  s. = sample standard deviation.

Test Statistic Value:

$$t = \frac{5.83 - 1.97}{\left[\frac{(4.52)^2}{15} + \frac{(3.99)^2}{48}\right]^{\frac{1}{2}}} = \frac{3.86}{1.30} = 2.97$$

$$d.f. = \frac{\left[\frac{(4.52)^2}{15} + \frac{(3.99)^2}{48}\right]^2}{\frac{\left[\frac{(4.52)^2}{15}\right]^2}{16} + \frac{\left[\frac{(3.99)^2}{48}\right]^2}{49}} - 2 =$$

$$= \frac{2.8686}{0.1182} - 2 \doteq 22$$

Result: Since $t_{.995} = 2.82$ for 22 d.f., the null hypothesis is not supported by the data, i.e., there is less than a one-half percent chance that random effects alone could have produced the value of $t = 2.97$ under the null hypothesis.

Null Hypothesis 3:

Mean nighttime concentrations at surface and near bottom in Michigan waters are equal.

Alternative: Concentration near bottom is greater.

Data:

$\bar{x}_S$	= 5.28	$\bar{x}_B$	= 18.83
s_S	= 11.18	s_B	= 16.13
n	= 30	n	= 30

Test Statistic Value:

$$t = \frac{18.83 - 5.28}{\left[\frac{(16.13)^2}{30} + \frac{(11.18)^2}{30}\right]^{\frac{1}{2}}} = \frac{13.55}{3.58} = 3.78$$

$$d.f. = \frac{164.84}{\frac{\left[\frac{(16.13)^2}{30}\right]^2}{31} + \frac{\left[\frac{(11.18)^2}{30}\right]^2}{31}} - 2 \doteq 53$$

Result: Since $t_{.995} = 2.70$ for 40 degrees of freedom and $t_{.995} = 2.66$ for 60 degrees of freedom, the null hypothesis is not supported by the data, i.e., there is less than a one-half percent chance that random effects alone could have produced the value of $t = 3.78$ under the null hypothesis. The observed value of t is highly significant.

Null Hypothesis 4:

Mean surface concentrations during daytime and nighttime in Michigan waters are equal.

Alternative: Surface concentration at nighttime is greater than surface concentration during daytime.

Data:

$\bar{x}_N$	= 5.28	$\bar{x}_D$	= 1.97
s_N	= 11.18	s_D	= 3.99
n	= 30	n	= 48

Test Statistic Value:

$$t = \frac{5.28 - 1.97}{\left[\frac{(11.18)^2}{30} + \frac{(3.99)^2}{48}\right]^{\frac{1}{2}}} = \frac{3.31}{2.12} = 1.56$$

$$d.f. = \frac{\left[\frac{(11.18)^2}{30} + \frac{(3.99)^2}{48}\right]^2}{\frac{\left[\frac{(11.18)^2}{30}\right]^2}{31} + \frac{\left[\frac{(3.99)^2}{48}\right]^2}{49}} - 2 = 35.99 - 2 = 34$$

Result: Since $t_{.90} = 1.31$ and 1.30 for 30 and 40 degrees of freedom, respectively, the null hypothesis is not supported by the data at the ten percent level of significance. However, there is no reason to reject the null hypothesis at the five percent level of significance since $t_{.95} = 1.70$ and 1.68 for 30 and 40 degrees of freedom, respectively.

Null Hypothesis 5:

Mean bottom concentrations during daytime and nighttime in Michigan waters are equal.

Alternative: Bottom concentration at nighttime is greater than bottom concentration during daytime.

Data:

$\bar{x}_N$	=	18.83	$\bar{x}_D$	=	5.83
s_N	=	16.13	s_D	=	4.52
n	=	30	n	=	15

Test Statistic Value:

$$t = \frac{18.83 - 5.83}{\left[\frac{(16.13)^2}{30} + \frac{(4.52)^2}{15}\right]^{\frac{1}{2}}} = \frac{13.00}{3.17} = 4.10$$

$$\text{d.f.} = \frac{\left[\frac{(16.13)^2}{30} + \frac{(4.52)^2}{15}\right]^2}{\frac{\left[\frac{(16.13)^2}{30}\right]^2}{31} + \frac{\left[\frac{(4.52)^2}{15}\right]^2}{16}} - 2 = 39.61 - 2 \doteq 38$$

Result: Since $t_{.995} = 2.75$ and 2.70 for 30 and 40 degrees of freedom, respectively, the data do not support the null hypothesis at the one-half percent level of significance. There is less than a one-half percent chance that random effects alone could produce the value $t = 4.10$ under the null hypothesis. The observed value of t is highly significant.

Null Hypothesis 6:

Mean daytime concentrations in Zone A of Ohio waters at the surface and bottom are equal.

Alternative: Concentration at the surface is greater than concentration at the bottom.

Data:

$\bar{x}_S$	=	14.15	$\bar{x}_B$	=	8.00
s_S	=	32.85	s_B	=	13.67
n	=	20	n	=	20

Test Statistic Value:

$$t = \frac{14.15 - 8.00}{\left[\frac{(32.85)^2}{20}\right]^2 + \left[\frac{(13.67)^2}{20}\right]} = \frac{6.15}{7.96} = 0.77$$

$$\text{d.f.} = \frac{\left[\frac{(32.85)^2}{20} + \frac{(13.67)^2}{20}\right]^2}{\left[\frac{\frac{(32.85)^2}{20}}{21}\right]^2 + \left[\frac{\frac{(13.67)^2}{20}}{21}\right]^2} - 2 = 28.06 - 2 = 26$$

Result: Since $t_{.80} = 0.856$ for 26 degrees of freedom, the data do not provide evidence for rejecting the null hypothesis at the 20 percent level of significance, that is, there is more than a twenty percent chance that random effects alone would produce a value of $t = 0.77$ under the null hypothesis. The observed value of t is not highly significant.

Null Hypothesis 7:

Mean daytime concentrations at the surface and bottom in Zone C of Ohio waters are equal.

Alternative: Daytime concentration at bottom is greater.

Data:

$\bar{x}_B$	= 38.31	$\bar{x}_S$	= 1.69
s_B	= 104.82	s_S	= 8.70
n	= 42	n	= 42

Test Statistic Value:

$$t = \frac{38.31 - 1.69}{\left[\frac{(104.82)^2}{42} + \frac{(8.70)^2}{42}\right]^{\frac{1}{2}}} = \frac{36.62}{16.23} = 2.26$$

$$\text{d.f.} = \frac{\left[\frac{(104.82)^2}{42} + \frac{(8.70)^2}{42}\right]^2}{\left[\frac{\frac{(104.82)^2}{42}}{43}\right]^2 + \left[\frac{\frac{(8.70)^2}{42}}{43}\right]^2} - 2 = 43.59 - 2 \doteq 42$$

Result: Since $t_{.975} = 2.02$ and 2.00 for 40 and 60 degrees of freedom, respectively, the data do not support the null hypothesis at the 2.5 percent level of significance, that is, there is less than a 2.5 percent chance that random effects alone would produce the value $t = 2.26$ under the null hypothesis.

Null Hypothesis 8:

Mean daytime concentrations at the surface and bottom in Zone D of Ohio waters are equal.

Alternative: Daytime concentration at bottom is greater.

Data:

$\bar{x}_B$	= 6.58	$\bar{x}_S$	= 0.51
s_B	= 10.64	s_S	= 0.90
n	= 36	n	= 37

Test Statistic Value:

$$t = \frac{6.58 - 0.51}{\left[\frac{(10.64)^2}{36} + \frac{(0.90)^2}{37}\right]^{\frac{1}{2}}} = \frac{6.07}{1.78} = 3.41$$

$$\text{d.f.} = \frac{\left[\frac{(10.64)^2}{36} + \frac{(0.90)^2}{37}\right]^2}{\frac{\left[\frac{(10.64)^2}{36}\right]^2}{37} + \frac{\left[\frac{(0.90)^2}{36}\right]^2}{37}} - 2 = 37.51 - 2 \doteq 36$$

Result: Since $t_{.995} = 2.75$ and 2.70 for 30 and 40 degrees of freedom, respectively, there is less than a 0.5 percent chance that random effects alone would produce a value of $t = 3.41$ under the null hypothesis. The test statistic is highly significant.

Null Hypothesis 9:

Mean daytime concentrations at the surface and bottom in Zone E of Ohio waters are equal.

Alternative: Daytime concentration at bottom is greater.

Data:

$\bar{x}_B$	= 9.03	$\bar{x}_S$	= 0.31
s_B	= 14.97	s_S	= 0.64
n	= 32	n	= 32

Test Statistic Value:

$$t = \frac{9.03 - 0.31}{\left[\frac{(14.97)^2}{32} + \frac{(0.64)^2}{32}\right]^{\frac{1}{2}}} = \frac{8.72}{2.65} = 3.29$$

$$\text{d.f.} = \frac{\left[\frac{(14.97)^2}{32} + \frac{(0.64)^2}{32}\right]^2}{\frac{\left[\frac{(14.97)^2}{32}\right]^2}{33} + \frac{\left[\frac{(0.64)^2}{32}\right]^2}{33}} - 2 \doteq 32$$

Result: The value of t = 3.29 is highly significant at the 0.5 percent level for 32 degrees of freedom, i.e., it is highly improbable that random effects alone would produce the observed value of t.

TABLE A.1. MEASURED CONCENTRATIONS OF LARVAL YELLOW PERCH IN MICHIGAN WATERS (1975).

	Day		Night	
Date	Surface	Near Bottom	Surface	Near Bottom
5/21	24.4, 33.8 20.8, 34.6, 32.5	46.5, 17.0, 5.7, 19.2, 15.0	-	-
5/22	0, 0, 0, 0, 0	40.7, 70.3, 24.5 26.1, 14.0	1.1, 0, 3.2, 8.4 0, 0, 3.1, 15.7, 0, 0, 1.0, 16.9	0, 1.0, 4.1, 8.9, 0, 1.0, 5.3, 5.4, 1, 1.1, 4.1, 9.8
5/23	0, 3.2, 0, 0	35.6, 29.1, 22.4 49.1, 20.3	1.1, 2.4, 1.1, 3.8, 1.1, 1.2, 8.9, 6.7, 0, 2.2, 4.1, 11.7	1, 4.2, 6.4, 9.9, 0, 1.0, 8.9, 7.2, 0, 0, 15.0, 18.9
6/16	2.9, 0, 2.8 0, 0	9.5, 17.4, 4.9, 14.2, 10.1	0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1.0, 0	0.9, 0, 0, 0, 0, 0, 0, 0, 1.7, 0, 0, 0
6/18	0, 0, 0, 0, 0	2.4, 7.3, 9.3 17.2, 15.9	-	-
6/19	0, 0, 0, 0, 0	8.7, 2.6, 10.0, 0, 0	0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0	0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
	$\bar{x}$ = 5.28 s = 11.18 n = 30	$\bar{x}$ = 18.83 s = 16.13 n = 30	$\bar{x}$ = 1.97 s = 3.99 n = 48	$\bar{x}$ = 2.43 s = 4.28 n = 48

Data Source: R.A. Cole, Institute of Water Research, Department of Fisheries and Wildlife, Michigan State University: Ref. (4).

TABLE A.2. FORMULAE FOR TESTING EQUALITY OF POPULATION MEANS

Null Hypothesis: Population Means Are Equal, Variances of Populations Assumed To Be Unknown and Not Necessarily Equal.

Test Statistic*:

$$t = \frac{x_1 - x_2}{\left[\frac{s_1^2}{m_1} + \frac{s_2^2}{m_2}\right]^{1/2}}$$

Distributed approximately as Student's t with:

$$= \frac{\left[\frac{s_1^2}{m_1} + \frac{s_2^2}{m_1}\right]^2}{\frac{\left(\frac{s_1^2}{m_1}\right)^2}{m_1 + 1} + \frac{\left(\frac{s_2^2}{m_2}\right)^2}{m_2 + 1}} - 2$$

= Degrees of Freedom (d.f.)

*Johnson & Leone, Statistics and Experimental Design in Engineering and the Physical Sciences, Vol. 1, p. 226, Wiley, 1964.

APPENDIX B

STATISTICAL TESTS OF SIGNIFICANCE FOR DIFFERENCES IN CONCENTRATIONS OF LARVAL YELLOW PERCH IN DEPTH ZONES IN MICHIGAN WATERS IN 1976

Data Source: Table 9

Date: April 26-29, 1976

Data:

Depth Zone	Sample Size m	Sample Mean $\bar{x}$	Sample Standard Deviation s
0'-6'	17	57.3	134.00
6'-12'	18	24.6	46.7
12'-18'	15	2.62	3.28
18'-24'	25	0.63	1.26
24'-30'	15	0.45	0.49

Null Hypothesis 1:

There is no significant difference in mean concentrations in the 0'-6' and 6'-12' depth zones during April 26-29, 1976.

Alternative: Mean concentration in 0'-6' zone is greater.

Test Statistic and d.f.:

Given in Appendix A, Table A.2

Test Statistic Value:

$$t = \frac{57.3 - 24.6}{\left[\frac{(134)^2}{17} + \frac{(46.7)^2}{18}\right]^{\frac{1}{2}}} = \frac{32.7}{34.3} = 0.95$$

$$\text{d.f.} = \frac{\left[\frac{(134)^2}{17} + \frac{(46.7)^2}{18}\right]^2}{\left(\frac{(134)^2}{17}\right)^2 + \left(\frac{(46.7)^2}{18}\right)^2} - 2 = \frac{1386261}{62752} - 2 \doteq 20$$

Result: Calculated value of t not significant at 10 percent level.

Null hypothesis is accepted.

One concludes that the 0'-6' and 6'-12' depth zones can be lumped for purposes of computing mean larval concentration in Michigan waters. The remaining depth zones are lumped into a second group.

APPENDIX C

CALCULATION OF MEAN CONCENTRATION AND STANDARD ERROR
FOR YELLOW PERCH LARVAE IN MICHIGAN WATERS IN 1976
Data Source: Table 9

$$\text{Mean concentration on specified day} = \frac{1}{V_T}(V_1 \bar{x}_1 + V_2 \bar{x}_2)$$

$$\text{Standard error: (S.E.)} = \frac{1}{V_T}\left(V_1^2 \frac{s_1^2}{n_1} + V_2^2 \frac{s_2^2}{n_2}\right)^{\frac{1}{2}}$$

where:

$\bar{x}_1$ = mean concentration in 0'-12' zone

n_1 = sample size in 0'-12' zone

s_1^2 = sample variance in 0'-12' zone

$\bar{x}_2$ = mean concentration in 12'-30' zone

n_2 = sample size in 12'-30' zone

s_2^2 = sample variance in 12'-30' zone

V_1 = volume in 0'-12' zone

V_2 = volume of 12'-30' zone

V_T = total volume

Day 118 (4/26-29)

$n_1 = 35$ $\quad$ $n_1 = 55$

$\bar{x}_1 = 40.5$ $\quad$ $\bar{x}_2 = 1.16$

$s_1 = 100$ $\quad$ $s_2 = 2.14$

$V_1 = 0.566 \times 10^8$ $\quad$ $V_2 = 4.41 \times 10^8$ $\quad$ $V_T = 4.976 \times 10^8$

$$\bar{x} = \frac{1}{4.976 \times 10^8} (0.566 \times 10^8 \times 40.5 + 4.41 \times 10^8 \times 1.16) = 5.63$$

$$S.E. = \frac{1}{4.976 \times 10^8} [(.566 \times 10^8)^2 \times \frac{(100)^2}{35}$$

$$+ (4.41 \times 10^8)^2 \times \frac{(2.14)^2}{55}]^{\frac{1}{2}} = 1.94$$

Mean concentration $\pm$ 1 S.E. = 5.63 $\pm$ 1.94 = 3.69, 7.57

Day 136

n_1	= 14	n_2	= 31
$\bar{x}_2$	= 0.562	$\bar{x}_2$	= 2.17
s_1	= 0.943	s_2	= 3.86

$$\bar{x} = \frac{1}{4.976 \times 10^8} (0.566 \times 10^8 \times 0.562 + 4.41 \times 10^8 \times 2.17) = 1.99$$

$$S.E. = \frac{1}{4.976 \times 10^8} [(.566 \times 10^8)^2 \times \frac{(.943)^2}{14}$$

$$+ (4.41 \times 10^8)^2 \times \frac{(3.86)^2}{31}]^{\frac{1}{2}} = 0.624$$

Mean concentration $\pm$ 1 S.E. = 1.99 $\pm$ 0.624 = 1.37, 2.61

Day 145

n_1	= 36	n_2	= 31
$\bar{x}_1$	= 3.94	$\bar{x}_2$	= 1.91
s_1	= 8.87	s_2	= 3.37

$$\bar{x} = \frac{1}{4.976 \times 10^8} (0.566 \times 10^8 \times 3.94 + 4.41 \times 10^8 \times 1.91) = 2.14$$

$$S.E. = \frac{1}{4.976 \times 10^8} [(.566 \times 10^8)^2 \times \frac{(8.87)}{36}$$

$$+ (4.41 \times 10^8)^2 \times \frac{(3.37)^2}{31}]^{\frac{1}{2}} = 0.56$$

Mean concentration $\pm$ 1 S.E. = 2.14 $\pm$ 0.56 = 1.58, 2.70

Day 158

$n_1 = 13$ $\quad n_2 = 20$

$\bar{x}_1 = 0.579$ $\quad \bar{x}_2 = 0.471$

$s_1 = 1.17$ $\quad s_2 = 0.996$

$$\bar{x} = \frac{1}{4.976 \times 10^8} (0.566 \times 10^8 \times 0.579 + 4.41 \times 10^8 \times 0.471) = 0.483$$

$$S.E. = \frac{1}{4.976 \times 10^8} [(0.566 \times 10^8)^2 \times \frac{(1.17)^2}{13}$$

$$+ (4.41 \times 10^8)^2 \times \frac{(0.996)^2}{20}]^{\frac{1}{2}} = 0.201$$

Mean concentration $\pm$ 1 S.E. = 0.483 $\pm$ 0.201 = 0.282, 0.684

Day 188

$n_1 = 15$ $\quad n_2 = 20$

$\bar{x}_1 = 0.133$ $\quad \bar{x}_2 = 0.147$

$s_1 = 0.225$ $\quad s_2 = 0.287$

$$\bar{x} = \frac{1}{4.976 \times 10^8} (0.566 \times 10^8 \times 0.133 + 4.41 \times 10^8 \times 0.147) = 0.145$$

$$S.E. = \frac{1}{4.976 \times 10^8} [(0.566 \times 10^8)^2 \times \frac{(0.225)^2}{15}$$

$$+ (4.41 \times 10^8)^2 \times \frac{(0.287)^2}{20}]^{\frac{1}{2}} = 0.057$$

Mean concentration $\pm$ 1 S.E. = 0.145 $\pm$ 0.057 = 0.088, 0.202

Day 201

$n_1 = 13$ $\quad n_2 = 8$

$\bar{x}_1 = 0.140$ $\quad \bar{x}_2 = 0.213$

$s_1 = 0.285$ $\quad s_2 = 0.562$

$$\bar{x} = \frac{1}{4.976 \times 10^8} (0.566 \times 10^8 \times 0.14 + 4.41 \times 10^8 \times 0.213) = 0.205$$

$$\text{S.E.} \frac{1}{4.976 \times 10^8} [(0.566 \times 10^8)^2 \times \frac{(0.285)^2}{} + (4.41 \times 10^8)^2 \times \frac{(0.562)^2}{8}]^{\frac{1}{2}} = 0.176$$

Mean concentration $\pm$ 1 S.E. = 0.205 $\pm$ 0.176 = 0.029, 0.381

Day 209

$n_1 = 5$ $\quad n_2 = 20$

$\bar{x}_1 = 0$ $\quad \bar{x}_2 = 0.126$

$s_1 = 0$ $\quad s_2 = 0.392$

$$\bar{x} = \frac{1}{4.976 \times 10^8} (4.41 \times 10^8 \times 0.126) = 0.112$$

$$\text{S.E.} \frac{1}{4.976 \times 10^8} [(4.41 \times 10^8)^2 \times \frac{(0.392)^2}{20}]^{\frac{1}{2}} = 0.078$$

Mean concentration $\pm$ 1 S.E. = 0.112 $\pm$ 0.078 = 0.034, 0.190

APPENDIX D

SAMPLE CALCULATION OF MEAN CONCENTRATIONS OF PROLARVAE (PROL), EARLY POSTLARVAE (EPL), AND LATE POSTLARVAE (LPL) IN MICHIGAN WATERS IN 1976

April 27-30 (Day 118)

Let C_T = total mean concentration of larvae in Michigan waters on day 118, 1976.

= 5.63

C_{PROL} = concentration of pro-larvae on day 118

= C_T x mean fraction PROL.

$$\text{Mean fraction PROL} = \frac{1}{497.6} (56.6 \times X_1 + 441 \times X_2)$$

where:

X_1 = fraction PROL in 0-12 ft. zone.

X_2 = fraction PROL in 12-30 ft. zone.

$$X_1 = \frac{2.17}{2.17 + 0.06 + 0} = 0.973$$

$$X_2 = \frac{1.29}{1.29 + 0 + 0} = 1.00$$

$$\text{Mean fraction PROL} = \frac{1}{497.6} (56.6 \times 0.973 + 441 \times 1.0)$$

= 0.997

C_{PROL} = 5.63 x 0.997 = 5.61

C_{EPL} = concentration of early post larvae on day 118.

= C_T x mean fraction EPL.

Mean fraction EPL $= \frac{1}{497.6} (56.6 \times y_1 + 441 \times y_2)$

where:

y_1 = fraction EPL in 0-12 ft. zone

y_2 = fraction EPL in 12-30 ft. zone

$y_1 = \frac{0.06}{2.17 + 0.06 + 0} = 0.027$

$y_2 = \frac{0}{1.29 + 0 + 0} = 0$

Mean fraction EPL $= \frac{1}{497.6} (56.6 \times 0.027 + 441 \times 0)$

$= 0.003$

$C_{EPL} = 5.63 \times 0.003 = .017$

C_{LPL} = concentration of late post larvae on day 118.

$= C_T$ x mean fraction LPL.

Mean fraction LPL $= \frac{1}{497.6} (56.6 \times Z_1 + 441 \times Z_2)$

where:

Z_1 = fraction LPL in 0-12 ft. zone.

Z_2 = fraction LPL in 12-30 ft. zone.

$Z_1 = \frac{0}{2.17 + 0.06 + 0} = 0$

$Z_2 = \frac{0}{1.29 + 0 + 0} = 0$

mean fraction LPL $= \frac{1}{497.6} (56.6 \times 0 + 441 \times 0) = 0$

$C_{LPL} = 5.63 \times 0 = 0$

APPENDIX E

ESTIMATING PERCENT MORTALITY OF ENTRAINED LARVAE

Method 1.

Let N_E = estimated number of live larvae entrained on a given day.

Let N_L = estimated number of live larvae entering upper discharge canal from plant discharge on the same day.

Then, estimated percent mortality, 100 p, on that day is:

$$100\ p = 100\ \frac{(N_E - N_L)}{N_E} = 100\ (1 - \frac{N_L}{N_E}) \qquad (E.1)$$

Equation E.1 requires knowledge of volume of cooling water on the given day.

The following method of estimating percent mortality on a given day can be applied using only knowledge of sample concentrations of live larvae entrained and discharged.

Method 2.

Let x_E = mean concentration of live larvae in cooling water entering plant on the given day.

Let x_L = mean concentration of live larvae entering upper discharge canal on the given day from plant discharge.

Then,

$$100\ p = 100\ (1 - \frac{x_L}{x_E}) \qquad (E.2)$$

Methods (1) and (2) defined above use concentrations of live larvae only and can be used only when $x_L < x_E$.

Method 3 which follows below contains an adjustment which permits the inclusion of counts of dead as well as live larvae which removes the restriction $x_L < x_E$.

Method 3.

Let x_L and x_E be defined as before. Let D_E and D_L denote the mean concentrations of dead larvae that are entrained and discharged from the power plant in the cooling water, respectively on the given day. Then percent mortality due to entrainment, 100 p, is:

$$100\ p = 100\ \left(1 - \frac{\frac{x_L}{x_L + D_L}}{\frac{x_E}{x_E + D_E}}\right) = 100\ \left(1 - \frac{x_L}{x_E} \cdot \frac{x_E + D_E}{x_L + D_L}\right) \qquad \text{(E.3)}$$

Equation (E.3) differs from (E.2) by an adjustment factor $\frac{x_E + D_E}{x_L + D_L}$ which utilizes counts of both live and dead larvae collected at the intake and outlet and also adjusts for different size samples collected at the intake and outlet.

A fourth method, similar to method 3, uses sample ratios of dead larvae to total larvae.

Method 4.

Let all variables be defined as given above. Then,

$$100\ p = 100\ \left(1 - \frac{\frac{D_E}{D_E + x_E}}{\frac{D_L}{D_L + x_L}}\right) = 100\ \left(1 - \frac{D_E}{D_L} \cdot \frac{D_L + x_L}{D_E + x_E}\right) \qquad \text{(E.4)}$$

Example

To illustrate the application of these four methods, consider the hypothetical data displayed in Table E.1. Following Table E.1 are the four calculations of estimated percent mortality of larvae due to entrainment which are 87.204, 86.489, 82.525, and 79.797 percent, respectively. Methods 1 and 2 are equivalent provided the mean concentrations in method 2 are calculated as shown in the example. The base population in both methods 1 and 2 is live larvae which is a subset of the total entrained population. Since methods 1 and 2 do not incorporate counts of dead larvae, information about

entrainment mortality is lost. For example, sampling variation may result in a low count of live larvae entering the plant which will lower the estimate of entrainment mortality. A high count of dead larvae at the discharge, however, indicates that there may have been substantial larval mortality as a result of entrainment. The inclusion of counts of dead as well as live larvae will use all the available information related to entrainment mortality. Methods 1 and 2 are modified (in methods 3 and 4) to incorporate both live and dead larval counts so that the base population is the entire entrained population for a given species. As noted by equation (E.3) the ratio of live larvae discharged to live larvae entrained is multiplied by an adjustment factor which incorporates counts of dead larvae that are entrained and discharged. The effect can be to either increase or decrease the estimated percentage of larval mortality due to entrainment. In the above hypothetical example the calculated percentage was reduced. Method 4 is similar to method 3 but the roles of dead and live larval counts are reversed. In method 4 dead larvae receive the same emphasis that live larvae received in method 3. Different percentages result, however, since counts of dead larvae entrained and discharged from the plant are different from counts of live larvae entrained and discharged from the plant. In the above example the estimated percentage of larval mortality resulting from entrainment given by method 4 is smaller than that given by method 3. In another example, the magnitudes of the percentages given by the two methods could be reversed. Just as a comparison of live larval counts before and after entrainment can be used as the basis for estimating mortality due to entrainment, a comparison of dead larval counts can be used in a manner exactly analogous, (methods 3 and 4) but which results in different numerical values for the estimate because the counts are not the same. There should be no theoretical reason why method 3 should be preferred over method 4 or vice versa. It is recommended, therefore, that the average of the two values be used as the estimate of larval mortality due to entrainment. In the above example, the estimated mortality due to entrainment is therefore, $1/2 \times (82.525 + 79.797) = 81.161$ percent. It is noted, incidentally, that the ratio of dead larvae to live plus dead larvae at the discharge is not a satisfactory method of estimating larvae mortality due to entrainment as it fails to compensate for larval mortality due to sampling.

TABLE E.1. COOLING WATER VOLUMES AND LARVAL CAPTURE DATA HYPOTHESIZED FOR EXAMPLE E.1.

Daytime hrs: 0500 - 2100 (5 a.m. - 9 p.m.)

Avg. cooling water inflow: 59.5 cu. meters per sec.

Vol. sampled (plant intake) 130 cu. meters		Vol. sampled (upper discharge) 75 cu. meters	
live pro larvae	15	live pro larvae	2
dead pro larvae	3	dead pro larvae	12
live post larvae	4	live post larvae	2
dead post larvae	1	dead post larvae	8

Nighttime hrs: 2100 - 0500 (9 p.m. - 5 a.m.)

Avg. cooling water inflows: 31.1 cu. meters per sec.

Vol. sampled (plant intake) 28 cu. meters		Vol. sampled (upper discharge) 48 cu. meters	
live pro larvae	63	live pro larvae	10
dead pro larvae	15	dead pro larvae	80
live post larvae	28	live post larvae	8
dead post larvae	4	dead post larvae	31

Calculations

Amt. of cooling water flowing into the plant in daytime from 0500 to 2100 = 3,404,800 cubic meters

Amt. of cooling water flowing into the plant in nighttime from 2100 to 0500 = 1,783,466 cubic meters

Calculation of N_E

live pro larvae (daytime) = $15 \times \frac{3404800}{130} = 392,861$

live post larvae (daytime) = $4 \times \frac{3404800}{130} = 104,763$

live pro larvae (nighttime) = $63 \times \frac{1783466}{28} = 4,012,798$

live post larvae (nighttime) = $28 \times \frac{1783466}{28} = 1,783,466$

$N_E = 6,293,888$

Calculation of N_L

live pro larvae (daytime) = $2 \times \frac{2404800}{75} = 90,794$

live post larvae (daytime) = $2 \times \frac{3404800}{75} = 90,794$

live pro larvae (nighttime) = $10 \times \frac{1783466}{48} = 371,555$

live post larvae (nighttime) = $8 \times \frac{1783466}{48} = 297,244$

$N_L = 805,387$

Method 1

$$100\ p = 100\ (1\ \frac{805387}{6293888}) = 100\ (1 - .12796)$$

$$= 87.204\%$$

Method 2

$$X_E = \frac{3404800}{3404800 + 1783466} \cdot (\frac{15 + 4}{1.3}) + \frac{1783466}{3404800 + 1783466} \cdot (\frac{63 + 28}{0.28})$$

$$= (.65625) \cdot (14.615) + (.34375) \cdot (325) =$$

$$9.5911 + 111.72 = 121.31$$

$$X_L = \frac{3404800}{3404800 + 1783466} \cdot (\frac{2 + 2}{.75}) + \frac{1783466}{3404800 + 1783466} \cdot (\frac{10 + 8}{0.48})$$

$$= (.65625)\ (5.3333) + (.34375)\ (37.5) =$$

$$3.4999 + 12.890 = 16.3899$$

$$100\ p = 100\ (1 - \frac{16.3899}{121.31}) = 100\ (1 - .13511) = 100\ (.86489)$$

$$= 86.489\%$$

Method 3

$$D_E = (.65625)\ (\frac{3 + 1}{1.3}) + (.34375)\ (\frac{15 + 4}{0.28}) = 2.0192 + 23.3256$$

$$= 25.345$$

$$D_L = (.65625)\ (\frac{12 + 8}{0.75}) + (.34375)\ (\frac{80 + 31}{0.48}) = 17.500 + 79.492$$

$$= 96.992$$

$$100\ p = 100\ (1 - \frac{X_L}{X_E} \cdot \frac{X_E + D_E}{X_L + D_L}) = 100\ (1 - 0.13511 \cdot \frac{121.31 + 25.345}{16.3899 + 96.992}) =$$

$$= 100\ (1 - (0.13511)\ (1.2934))$$

$$= 100\ (1 - .179\ 75)$$

$$82.525\%$$

Method 4

$$100\ p = 100\ \left(1 - \frac{25.345}{96.992} \cdot \frac{96.992 + 16.389}{25.345 + 121.31}\right)$$

$$= 100\ (1 - .2613\ (.773))$$

$$= 79.80\%$$

APPENDIX F

SOLUTIONS TO FIRST ORDER EQUATIONS OF LARVAL BALANCE FOR MICHIGAN AND OHIO WATERS, 1975 AND 1976

Given:

$$\dot{N}(t) = - p\, N(t) + h(t) + f(t) + (h(t-25) + f(t-25))e^{-25p} - E(t) \qquad (F.1)$$

$$(t \geq t_o > 0)$$

Equation (F.1) is of the form:

$$\dot{N}(t) + C\, N = F(t) \quad (t \geq t_o > 0). \qquad (F.2)$$

where:

C = constant;

$F(t)$ = function of time.

The general solution to (F.2) is:

$$N(t) = N(t_o)e^{-C\cdot(t-t_o)} + e^{-C\cdot(t-t_o)} \int_{t_o}^{t} F(Z)\, e^{C\cdot(Z-t_o)}\, dz \qquad (F.3)$$

$$(t \geq t_o)$$

and therefore (F.1) has the solution given by (F.3).

If $F(t)$ happens to be independent of the parameter t, say $F(t) = F$, then (F.3) becomes:

$$N(t) = N(t_o)e^{-C\cdot(t-t_o)} + \frac{F}{C}\,(1 - e^{-C\cdot(t-t_o)}) \qquad (F.4)$$

$$(t \geq t_o)$$

If the definitions of the parameter C or the function $F(t)$ are specific to subintervals of time as are the cases represented by Equations (7), (8), (9), and (10) above, then for subinterval i, let $C = C_i$ and $F = F_i$ $(i = 1, \ldots, n)$. Equation (F.4) is:

$$N(t) = \begin{cases} N(t_o)\cdot e^{-C_1(t-t_o)} + \dfrac{F_1}{C_1}\cdot\left(1 - e^{-C_1\cdot(t-t_o)}\right) & (t_o < t \le t_1) \\ (Nt_1)\cdot e^{-C_2\cdot(t-t_1)} + \dfrac{F_2}{C_2}\cdot\left(1 - e^{-C_2\cdot(t-t_1)}\right) & (t_1 < t \le t_2) \\ N(t_2)\cdot e^{-C_3\cdot(t-t_2)} + \dfrac{F_3}{C_3}\cdot\left(1 - e^{-C_3\cdot(t-t_2)}\right) & (t_2 < t \le t_3) \\ \quad\vdots & \quad\vdots \\ N(t_{n-1})\cdot e^{-C_n(t-t_{n-1})} + \dfrac{F_n}{C_n}\cdot\left(1 - e^{-C_n(t-t_{n-1})}\right) & (t_{n-1} < t \le t_n) \end{cases} \qquad (F.5)$$

Writing down the balances defined by Equation (F.1) for each of the cases represented by Equations (7)-(10) above, the following differential equations of balance and their solutions are obtained.

Michigan Water, 1976 (using Equation 10)

$$\frac{1}{B}[\dot{N}(t) + p \cdot N(t)] = \begin{cases} 0 & 0 < t \leq 106 \\ 0.4437 \cdot h & 106 < t \leq 120 \\ 0.3915 \cdot h & 120 < t \leq 131 \\ (0.3915 - 0.4437\ e^{-25p}) \cdot h & 131 < t \leq 134 \\ (0.1382 - 0.4437\ e^{-25p}) \cdot h & 134 < t \leq 145 \\ (0.1382 - 0.3915\ e^{-25p}) \cdot h & 145 < t \leq 148 \\ (0.0244 - 0.3915\ e^{-25p}) \cdot h & 148 < t \leq 159 \\ (0.0244 - 0.1382\ e^{-25p}) \cdot h & 159 < t \leq 162 \\ (0.0022 - 0.1382\ e^{-25p}) \cdot h & 162 < t \leq 173 \\ (0.0022 - 0.0244\ e^{-25p}) \cdot h & 173 < t \leq 176 \\ (0.0001 - 0.0244\ e^{-25p}) \cdot h & 176 < t \leq 187 \\ (0.0001 - 0.0022\ e^{-25p}) \cdot h & 187 < t \leq 189 \\ -0.0022 \cdot h \cdot e^{-25p} & 189 < t \leq 200 \\ -0.0001 \cdot h \cdot e^{-25p} & 200 < t \leq 214 \\ 0 & 214 < t \leq 365 \end{cases}$$

$$(B = 497.6 \times 10^{4};\ E(t) = 0) \qquad \text{(F.6)}$$

The solution to Equation (F.6) following the format of Equation (F.5) is:

$$N(t) = 0 \qquad 0 < t \leq 106$$

$$N(t) = N(106) \cdot e^{-p(t-106)} + 0.4437\ (\frac{h \cdot B}{p})(1 - e^{-p(t-106)})$$

$$106 < t \leq 120$$

$$N(t) = N(120) \cdot e^{-p(t-120)} + 0.3915\ (\frac{h \cdot B}{p})(1 - e^{-p(t-106)})$$

$$120 < t \leq 131$$

$$N(t) = N(131)\cdot e^{-p(t-131)} + (0.3915 - 0.4437\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-131)})$$

$$131 < t \leq 134$$

$$N(t) = N(134)\cdot e^{-p(t-134)} + (0.1382 - 0.4437\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-134)})$$

$$134 < t \leq 145$$

$$N(t) = N(145)\cdot e^{-p(t-145)} + (0.1382 - 0.3915\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-145)})$$

$$145 < t \leq 148$$

$$N(t) = N(148)\cdot e^{-p(t-148)} + (0.0244 - 0.3915\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-148)})$$

$$148 < t \leq 159$$

$$N(t) = N(159)\cdot e^{-p(t-159)} + (0.0244 - 0.1382\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-159)})$$

$$159 < t \leq 162$$

$$N(t) = N(162)\cdot e^{-p(t-162)} + (0.0022 - 0.1382\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-162)}$$

$$162 < t \leq 173$$

$$N(t) = N(173)\cdot e^{-p(t-173)} + (0.0022 - 0.0244\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-173)})$$

$$173 < t \leq 176$$

$$N(t) = N(176)\cdot e^{-p(t-176)} + (0.0001 - 0.0244\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-176)})$$

$$176 < t \leq 187$$

$$N(t) = N(187)\cdot e^{-p(t-187)} + (0.0001 - 0.0022\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-187)})$$

$$187 < t \leq 189$$

$$N(t) = N(189)\cdot e^{-p(t-189)} + 0.0022\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-189)})$$

$$189 < t \leq 200$$

$$N(t) = N(200)\cdot e^{-p(t-200)} + 0.0001\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-200)}\right)$$

$$200 < t \leq 214$$

$$N(t) = N(214)\cdot e^{-p(t-214)}$$

$$214 < t \leq 365 \qquad \text{(F.7)}$$

Ohio Waters, 1975 (using Equation 7)

$$\frac{1}{B}[\dot{N}(t) + p\cdot N(t)] = \begin{cases} 0 & 0 < t \leq 127 \\ 0.5905\cdot h & 127 < t \leq 134 \\ 0.328\cdot h & 134 < t \leq 141 \\ 0.0729\cdot h & 141 < t \leq 148 \\ 0.0081\cdot h & 148 < t \leq 152 \\ (0.0081 - 0.5905\cdot e^{-25p})\cdot h & 152 < t \leq 155 \\ (0.0004 - 0.5905\cdot e^{-25p})\cdot h & 155 < t \leq 159 \\ (0.0004 - 0.328\cdot e^{-25p})\cdot h & 159 < t \leq 162 \\ -0.328\cdot e^{-25p}\cdot h & 162 < t \leq 166 \\ -0.0729\cdot e^{-25p}\cdot h & 166 < t \leq 173 \\ -0.0081\cdot e^{-25p}\cdot h & 173 < t \leq 180 \\ -0.0004\cdot e^{-25p}\cdot h & 180 < t \leq 187 \\ 0 & 187 < t \leq 365 \end{cases}$$

$$(B = 9.393 \times 10^{7};\ E(t) = 0) \qquad \text{(F.8)}$$

The solution to Equation (F.8) following the format of Equation (F.5) is:

$$N(t) = 0 \qquad 0 < t \leq 127$$

$$N(t) = (0.5905)\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-127)}\right)$$

$$127 < t \leq 134$$

$$N(t) = N(134)\cdot e^{-p(t-134)} + (0.328)\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-134)}\right)$$

$$134 < t \leq 141$$

$$N(t) = N(141)\cdot e^{-p(t-141)} + (0.0729)\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-141)}\right)$$

$$141 < t \leq 148$$

$$N(t) = N(148)\cdot e^{-p(t-148)} + (0.0081)\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-148)}\right)$$

$$148 < t \leq 152$$

$$N(t) = N(152)\cdot e^{-p(t-152)} + (0.0081 - 0.5905\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-152)}\right)$$

$$152 < t \leq 155$$

$$N(t) = N(155)\cdot e^{-p(t-155)} + (0.0004 - 0.5905\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-155)}\right)$$

$$155 < t \leq 159$$

$$N(t) = N(159)\cdot e^{-p(t-159)} + (0.0004 - 0.328\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-159)}\right)$$

$$159 < t \leq 162$$

$$N(t) = N(162)\cdot e^{-p(t-162)} - 0.328\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-162)}\right)$$

$$162 < t \leq 166$$

$$N(t) = N(166)\cdot e^{-p(t-166)} - 0.0729\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-166)}\right)$$

$$166 < t \leq 173$$

$$N(t) = N(173)\cdot e^{-p(t-173)} - 0.0081\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-173)}\right)$$

$$173 < t \leq 180$$

$$N(t) = N(180)\cdot e^{-p(t-180)} - 0.0004\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)\left(1-e^{-p(t-180)}\right)$$

$$180 < t \leq 187$$

$$N(t) = N(187)\cdot e^{-p(t-187)}$$

$$187 < t \leq 365 \qquad \text{(F.9)}$$

Ohio Waters, 1976 (using Equation 8)

$$\frac{1}{B}[\dot{N}(t) + p \cdot N(t)] = \begin{cases} 0 & 0 < t \leq 106 \\ 0.5905 \cdot h & 106 < t \leq 113 \\ 0.328 \cdot h & 113 < t \leq 120 \\ 0.0729 \cdot h & 120 < t \leq 127 \\ 0.0081 \cdot h & 127 < t \leq 131 \\ (0.0081 - 0.5905 \cdot e^{-25p}) \cdot h & 131 < t \leq 134 \\ (0.0004 - 0.5905 \cdot e^{-25p}) \cdot h & 134 < t \leq 138 \\ (0.0004 - 0.328 \cdot e^{-25p}) \cdot h & 138 < t \leq 141 \\ - 0.328 \cdot e^{-25p} \cdot h & 141 < t \leq 145 \\ - 0.0729 \cdot e^{-25p} \cdot h & 145 < t \leq 152 \\ - 0.0081 \cdot e^{-25p} \cdot h & 152 < t \leq 159 \\ - 0.0004 \cdot e^{-25p} \cdot h & 159 < t \leq 166 \\ 0 & 166 < t \leq 365 \end{cases}$$

$$(B = 9.393 \times 10^{7};\ E(t) = 0) \qquad \text{(F.10)}$$

The solution to Equation (F.10) following the format of Equation (F.5) is:

$$N(t) = 0 \qquad 0 < t \leq 106$$

$$N(t) = (0.5905)\left(\frac{h \cdot B}{p}\right)(1 - e^{-p(t-106)}) \qquad 106 < t \leq 113$$

$$N(t) = N(113) \cdot e^{-p(t-113)} + (0.328)\left(\frac{h \cdot B}{p}\right)(1 - e^{-p(t-113)}) \qquad 113 < t \leq 120$$

$$N(t) = N(120) \cdot e^{-p(t-120)} + (0.0729)\left(\frac{h \cdot B}{p}\right)(1 - e^{-p(t-120)}) \qquad 120 < t \leq 127$$

$$N(t) = N(127) \cdot e^{-p(t-127)} + (0.0081)\left(\frac{h \cdot B}{p}\right)(1 - e^{-p(t-127)}) \qquad 127 < t \leq 131$$

$$N(t) = N(131)\cdot e^{-p(t-131)} + (0.0081 - 0.5905\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-131)})$$

$$131 < t \leq 134$$

$$N(t) = N(134)\cdot e^{-p(t-134)} + (0.0004 - 0.5905\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-134)})$$

$$134 < t \leq 138$$

$$N(t) = N(138)\cdot e^{-p(t-138)} + (0.0004 - .328\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-138)})$$

$$138 < t \leq 141$$

$$N(t) = N(141)\cdot e^{-p(t-141)} - 0.328\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-141)})$$

$$141 < t \leq 145$$

$$N(t) = N(145)\cdot e^{-p(t-145)} - 0.0729\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-145)})$$

$$145 < t \leq 152$$

$$N(t) = N(152)\cdot e^{-p(t-152)} - 0.0081\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-152)})$$

$$152 < t \leq 159$$

$$N(t) = N(159)\cdot e^{-p(t-159)} - 0.0004\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-159)})$$

$$159 < t \leq 166$$

$$N(t) = N(166)\cdot e^{-p(t-166)}$$

$$166 < t \leq 365 \qquad \text{(F.11)}$$

Michigan Water, 1975 (using Equation 9)

$$\frac{1}{B}[\dot{N}(t) + p \cdot N(t)] = \begin{cases} 0 & 0 < t \leq 120 \\ 0.4437 \cdot h & 120 < t \leq 134 \\ 0.3915 \cdot h & 134 < t \leq 145 \\ (0.3915 - 0.4437 \cdot e^{-25p}) \cdot h & 145 < t \leq 148 \\ (0.1382 - 0.4437 \cdot e^{-25p}) \cdot h & 148 < t \leq 159 \\ (0.1382 - 0.3915 \cdot e^{-25p}) \cdot h & 159 < t \leq 162 \\ (0.0244 - 0.3915 \cdot e^{-25p}) \cdot h & 162 < t \leq 173 \\ (0.0244 - 0.1382 \cdot e^{-25p}) \cdot h & 173 < t \leq 176 \\ (0.0001 - 0.1382 \cdot e^{-25p}) \cdot h & 176 < t \leq 187 \\ (0.0001 - 0.0244 \cdot e^{-25p}) \cdot h & 187 < t \leq 189 \\ - 0.0244 \cdot e^{-25p} \cdot h & 189 < t \leq 201 \\ - 0.0001 \cdot e^{-25p} \cdot h & 201 < t \leq 215 \\ 0 & 215 < t \leq 365 \end{cases}$$

$$(B = 497.6 \times 10^4;\ E(t) = 0) \qquad \text{(F.12)}$$

The solution to Equation (F.12) following the format of Equation (F.5) is:

$$N(t) = 0 \qquad 0 < t \leq 120$$

$$N(t) = 0.4437 \left(\frac{h \cdot B}{p}\right)\left(1 - e^{-p\,(t-120)}\right)$$

$$120 < t \leq 134$$

$$N(t) = N(134) \cdot e^{-p(t-134)} + 0.3915 \left(\frac{h \cdot B}{p}\right)\left(1 - e^{-p(t-134)}\right)$$

$$134 < t \leq 145$$

$$N(t) = N(145) \cdot e^{-p(t-145)} + (0.3915 - 0.4437 \cdot e^{-25p})\left(\frac{h \cdot B}{p}\right)\left(1 - e^{-p(t-145)}\right)$$

$$145 < t \leq 148$$

$$N(t) = N(148) \cdot e^{-p(t-148)} + (0.1382 - 0.4437 \cdot e^{-25p})\left(\frac{h \cdot B}{p}\right)\left(1 - e^{-p(t-148)}\right)$$

$$148 < t \leq 159$$

$$N(t) = N(159)\cdot e^{-p(t-159)} + (0.1382 - 0.3915\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-159)})$$

$$159 < t \leq 162$$

$$N(t) = N(162)\cdot e^{-p(t-162)} + (0.0244 - 0.3915\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-162)})$$

$$162 < t \leq 173$$

$$N(t) = N(173)\cdot e^{-p(t-173)} + (0.0244 - 0.1382\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-173)})$$

$$173 < t \leq 176$$

$$N(t) = N(176)\cdot e^{-p(t-176)} + (0.0001 - 0.1382\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-176)})$$

$$176 < t \leq 187$$

$$N(t) = N(187)\cdot e^{-p(t-187)} + (0.0001 - 0.0244\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-187)})$$

$$187 < t \leq 189$$

$$N(t) = N(189)\cdot e^{-p(t-189)} - 0.0244\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-189)})$$

$$189 < t \leq 201$$

$$N(t) = N(201)\cdot e^{-p(t-201)} - (0.0001)\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-201)})$$

$$201 < t \leq 215$$

$$N(t) = N(215)\cdot e^{-p(t-215)}$$

$$215 < t \leq 365 \qquad \text{(F.13)}$$

Michigan Water, 1975 (using Equations 9 and 14)

$$\dot{[N(t) + p \cdot N(t)]} = \begin{cases} 0 & 0 < t \leq 120 \\ 0.4437 \cdot h \cdot B & 120 < t \leq 125 \\ 0.4437 \cdot h \cdot B - 0.027 & 125 < t \leq 132 \\ 0.4437 \cdot h \cdot B - 0.053 & 132 < t \leq 134 \\ 0.3915 \cdot h \cdot B - 0.053 & 134 < t \leq 141 \\ 0.3915 \cdot h \cdot B - 0.025 & 141 < t \leq 145 \\ (0.3915 - 0.4437 \cdot e^{-25p}) \cdot h \cdot B - 0.025 & 145 < t \leq 148 \\ (0.1382 - 0.4437 \cdot e^{-25p}) \cdot h \cdot B - 0.013 & 148 < t \leq 156 \\ (0.1382 - 0.4437 \cdot e^{-25p}) \cdot h \cdot B - 0.004 & 156 < t \leq 159 \\ (0.1382 - 0.3915 \cdot e^{-25p}) \cdot h \cdot B - 0.004 & 159 < t \leq 162 \\ (0.0244 - 0.3915 \cdot e^{-25p}) \cdot h \cdot B - 0.004 & 162 < t \leq 170 \\ (0.0244 - 0.3915 \cdot e^{-25p}) \cdot h \cdot B & 170 < t \leq 173 \\ (0.0244 - 0.1382 \cdot e^{-25p}) \cdot h \cdot B & 173 < t \leq 176 \\ (0.0001 - 0.1382 \cdot e^{-25p}) \cdot h \cdot B & 176 < t \leq 187 \\ (0.0001 - 0.0244 \cdot e^{-25p}) \cdot h \cdot B & 187 < t \leq 189 \\ - 0.0244 \cdot e^{-25p} \cdot h \cdot B & 189 < t \leq 201 \\ - 0.0001 \cdot e^{-25p} \cdot h \cdot B & 201 < t \leq 215 \\ 0 & 215 < t \leq 365 \end{cases}$$

$(B = 497.6 \times 10^4$; E(t) given by Equation 14) (F.14)

The solution to Equation (F.14) following the format of Equation (F.5) is:

$$N(t) = 0 \qquad 0 < t \leq 120$$

$$N(t) = 0.4437 \frac{h \cdot B}{p} (1 - e^{-p(t-120)})$$

$$120 < t \leq 125$$

$$N(t) = N(125) \cdot e^{-p(t-125)} + (0.4437 - \frac{0.027}{h \cdot B})(\frac{h \cdot B}{p})(1 - e^{-p(t-125)})$$

$$125 < t \leq 132$$

$$N(t) = N(132)\cdot e^{-p(t-132)} + (0.4437 - \frac{0.053}{h\cdot B})(\frac{h\cdot B}{p})(1-e^{-p(t-132)})$$

$$132 < t \leq 134$$

$$N(t) = N(134)\cdot e^{-p(t-134)} + (0.3915 - \frac{0.053}{h\cdot B})(\frac{h\cdot B}{p})(1-e^{-p(t-134)})$$

$$134 < t \leq 141$$

$$N(t) = N(141)\cdot e^{-p(t-141)} + (0.3915 - \frac{0.025}{h\cdot B})(\frac{h\cdot B}{p})(1-e^{-p(t-141)})$$

$$141 < t \leq 145$$

$$N(t) = N(145)\cdot e^{-p(t-145)} + (0.3915 - 0.4437\cdot e^{-25p} - \frac{0.025}{h\cdot B})(\frac{h\cdot B}{p})(1-e^{-p(t-145)})$$

$$145 < t \leq 148$$

$$N(t) = N(148)\cdot e^{-p(t-148)} + (0.1382 - 0.4437\cdot e^{-25p} - \frac{0.013}{h\cdot B})(\frac{h\cdot B}{p})(1-e^{-p(t-148)})$$

$$148 < t \leq 156$$

$$N(t) = N(156)\cdot e^{-p(t-156)} + (0.1382 - 0.4437\cdot e^{-25p} - \frac{0.004}{h\cdot B})(\frac{h\cdot B}{p})(1-e^{-p(t-156)})$$

$$156 < t \leq 159$$

$$N(t) = N(159)\cdot e^{-p(t-159)} + (0.1382 - 0.3915\cdot e^{-25p} - \frac{0.004}{h\cdot B})(\frac{h\cdot B}{p})(1-e^{-p(t-159)})$$

$$159 < t \leq 162$$

$$N(t) = N(162)\cdot e^{-p(t-162)} + (0.0244 - 0.3915\cdot e^{-25p} - \frac{0.004}{h\cdot B})(\frac{h\cdot B}{p})(1-e^{-p(t-162)})$$

$$162 < t \leq 170$$

$$N(t) = N(170)\cdot e^{-p(t-170)} + (0.0244 - 0.3915\cdot e^{-25p})(\frac{h\cdot B}{p})(1-e^{-p(t-170)})$$

$$170 < t \leq 173$$

$$N(t) = N(173)\cdot e^{-p(t-173)} + (0.0244 - 0.1382\cdot e^{-25p})(\frac{h\cdot B}{p})(1-e^{-p(t-173)})$$

$$173 < t \leq 176$$

$$N(t) = N(176)\cdot e^{-p(t-176)} + (0.0001 - 0.1382\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-176)})$$

$$176 < t \leq 187$$

$$N(t) = N(187)\cdot e^{-p(t-187)} + (0.0001 - 0.0244\cdot e^{-25p})\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-187)})$$

$$187 < t \leq 189$$

$$N(t) = N(189)\cdot e^{-p(t-189)} - 0.0244\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-189)})$$

$$189 < t \leq 201$$

$$N(t) = N(201)\cdot e^{-p(t-201)} - 0.0001\cdot e^{-25p}\left(\frac{h\cdot B}{p}\right)(1-e^{-p(t-201)})$$

$$201 < t \leq 215$$

$$N(t) = N(215)\cdot e^{-p(t-215)}$$

$$215 < t \leq 365 \qquad \text{(F.15)}$$

APPENDIX G

APPROXIMATE VARIANCE OF EQUILIBRIUM POPULATION AS A FUNCTION OF REPRODUCTIVE POTENTIAL AND LARVAL SURVIVAL

Using Taylor's Expansion, one approximates Var(B) as:

$$\mathrm{Var}(B) = \left(\frac{\partial B}{\partial \varepsilon}\right)^2 \sigma_\varepsilon^2 + \left(\frac{\partial B}{\partial \gamma}\right)^2 \sigma^2$$

where the derivatives are evaluated at $\gamma = 50$ and $\varepsilon = .08$.

Let $\varepsilon = .08$, $s = .26$, $m = .37$, $\gamma = 50$

$K = 5 \times 10^7$, $E_\ell = 1 \times 10^7$, $E_y = 250{,}000$

$\sigma_\varepsilon = .04$, $\sigma_\gamma = 25$

Now:

$$B = \frac{1}{2}\left\{K + \frac{E_\ell}{\gamma} - \frac{(m+f)K}{\alpha s \varepsilon \gamma} + \left[\left(K + \frac{E_\ell}{\gamma} - \frac{(m+f)K}{\alpha s \varepsilon \gamma}\right)^2 - 4K\left(\frac{E_\ell}{\gamma} + \frac{E_y}{\gamma}\right)\right]^{\frac{1}{2}}\right\}$$

Then:

$$\frac{\partial B}{\partial \varepsilon} = \frac{1}{2}\left\{-\frac{(m+f)K}{\alpha s \varepsilon \gamma} \cdot \left(-\frac{1}{\varepsilon^2} + \frac{1}{2}\left[\left(K + \frac{E_\ell}{\gamma} - \frac{(m+f)K}{\alpha s \varepsilon \gamma}\right)^2 - 4K\left(\frac{E_\ell}{\gamma} + \frac{E_y}{\gamma}\right)\right]^{-\frac{1}{2}}\right.\right.$$

$$\left.\cdot \left[2\left(K + \frac{E_\ell}{\gamma} - \frac{(m+f)K}{\alpha s \varepsilon \gamma}\right)\left(\frac{(m+f)K}{\alpha s \varepsilon \gamma^2}\right) - \right.\right.$$

$$\left.\left. - 4K\left(-\frac{E_y}{\gamma \varepsilon^2}\right)\right]\right\}$$

$$\frac{\partial B}{\partial \gamma} = \frac{1}{2}\left[-\frac{E_\ell}{\gamma^2} + \frac{(m+f)K}{s^2} + \frac{1}{2}\left[\left(K + \frac{E_\ell}{\gamma} - \frac{(m+f)K}{\alpha s \varepsilon \gamma}\right)^2 - 4K\left(\frac{E_\ell}{\gamma} + \frac{E_y}{\varepsilon \gamma}\right)\right]^{-\frac{1}{2}} \cdot\right.$$

$$\cdot \left[2\left(K + \frac{E_\ell}{\gamma} - \frac{(m+f)K}{\alpha s \varepsilon \gamma}\right)\left(-\frac{E_\ell}{\gamma^2} + \frac{(m+f)K}{\alpha s \varepsilon \gamma^2}\right) - \right.$$

$$\left. - 4K\left(-\frac{E_\ell}{\gamma^2} - \frac{E_y}{\varepsilon\gamma^2}\right)\right]\}$$

$$\frac{\partial B}{\partial \varepsilon} = \frac{1}{2} + \frac{.74}{12.922} \cdot \frac{5 \cdot 10^7}{.0064}$$

$$+ \frac{1}{2}\left[\left(5 \cdot 10^7 + \frac{10^7}{50} - \frac{.74 \cdot 10^7}{1.03376}\right)^2 - 20 \cdot 10^7 \left(\frac{10^7}{50} + \frac{2.5 \cdot 10^5}{4}\right)\right]^{-\frac{1}{2}}$$

$$\cdot \left[\left(2(10^7 \cdot 4.3041665)\left(\frac{(.74)\ 5 \cdot 10^7}{.0827008} + \frac{4 \cdot 5 \cdot 10^7 \cdot 2.5 \cdot 10^5}{.32}\right)\right\}\right.$$

$$= \frac{1}{2}\ \ 44.739\text{-}9 \cdot 10^7 + \frac{1}{2}\left(10^{14}(18.525849) - 10^{14}(.525)\right]^{-\frac{1}{2}}$$

$$\cdot \left[10^{14}\ (385.1383 + 10^{14}\ (1.5625)\right]$$

$$= \frac{1}{2}\ 44.73959 \cdot 10^7 + \frac{10^{14}(386.6958)}{2 \cdot 10^7 \cdot 4.242740} = 45.155526 \cdot 10^7$$

$$\left(\frac{\partial B}{\partial \varepsilon}\right)^2 = 10^{14}\ \ 2039.0215$$

$$\boxed{\left(\frac{\partial B}{\partial \varepsilon}\right)^2 \sigma_\varepsilon^2 = 10^{14}\ \ 3.262434}$$

$$\sigma_\varepsilon^2 = (.04)^2 = .0016$$

$$\frac{\partial B}{\partial \varepsilon} = \frac{1}{2}\left\{-\frac{10^7}{2500} + \frac{.74 \cdot 5 \cdot 10^7}{51.688} + \frac{1}{2}\ \ \left(5 \cdot 10^7 + \frac{10^7}{50} - \frac{.74 \cdot 5 \cdot 10^7}{1.03376}\right)^2\right.$$

$$\left. - 20 \cdot 10^7\ \left(\frac{10^7}{50} + \frac{.025 \cdot 10^7}{4}\right)\right]^{-\frac{1}{2}}$$

$$\cdot \left[2\left(5 \cdot 10^7 + \frac{10^7}{50} - \frac{.74 \cdot 5 \cdot 10^7}{1.03376}\right) \cdot \left(-\frac{10^7}{2500} + \frac{.74 \cdot 5 \cdot 10^7}{51.688}\right)\right.$$

$$+ 4 \cdot 5 \cdot 10^7 \left(\frac{10^7}{2500} + \frac{.025 \cdot 10^7}{200}\right)]\}$$

$$\frac{\partial B}{\partial \gamma} = \frac{1}{2}\ 10^7(.07118334) + \frac{1}{2}\ [10^{14}(2.0759988 - 10^{14}(.525)]^{-\frac{1}{2}}$$

$$\cdot\ [10^{14} \cdot (2.8816654) \cdot (.071183346) + 10^{14}(.0105)]\} =$$

$$= \frac{1}{2}\ [10^7(.07118334) + \frac{.21562658 \cdot 10^{14}}{10^7 \cdot 622645511}] = .20873133 \cdot 10^7$$

$$\left(\frac{\partial}{\partial \gamma}\right)^2 = 10^{14}(.043568768)$$

$$\sigma_\varepsilon^2 = (25)^2 = 625$$

$$\boxed{\left(\frac{\partial B}{\partial \gamma}\right)^2\ \sigma_\gamma^2 = 10^{14} \cdot 27.23042}$$

$$\text{Var}(B) = 10^{14}(27.23048 + 3.262434)$$

$$= 10^{14}(30.493214)$$

$$\text{STD. DEV. } B = 10^7 \cdot 5.52206609 = 55{,}220{,}661$$

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APPENDIX H

RELATIONSHIP BETWEEN AGE OF LARVAE AT ENTRAINMENT AND REDUCTION OF YOUNG OF YEAR POPULATION DUE TO ENTRAINMENT

It is estimated above that the fraction of larvae that survive natural mortality for 25 days and hence are recruited into the young-of-year population is between 2 and 10 percent of total production. Therefore, the fraction of larvae lost due to entrainment mortality that would be expected to have survived to reach the young-of-year stage would also be between 2 and 10 percent provided they are in their first day of life at the time of entrainment. At the other extreme all larvae which are in their 25th day of life at the time of entrainment would have survived to reach young-of-year stage, by definition, since they are at that stage at the time of entrainment. The fraction of larvae which are at some intermediate age at the time of entrainment that would be expected to survive to reach young-of-year is estimated as follows.

Define the following variables and functions.

X_1 = a geometrically distributed random variable with parameter p_1, defined on the positive integers, which denotes the age (in days) of larvae upon entering the reference volume.

$h_1(x_1)$ = probability function of X_1.

$$= \begin{cases} p_1(1 - p_1)^{x_1 - 1} & x_1 = 1,2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

X_2 = a geometrically distributed random variable with parameter p_2, defined on the positive integers, which denotes the number of days that larvae are in residence in the reference volume upon entering the entrainment cycle.

$h_2(x_2)$ = probability function of X_2.

$$= \begin{cases} p_2(1 - p_2)^{x_2 - 1} & x_2 = 1,2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

$Y = X_1 + X_2 - 1$ = a random variable denoting the age (in days) of larvae upon entering the entrainment cycle.

$g(y)$ = probability function of Y.

One easily verifies that:

$$g(y) = \begin{cases} \frac{p_1 p_2}{p_1 - p_2} (1 - p_2)^y - (1 - p_1)^y & y = 1,2, \ldots \\ 0 & \text{otherwise} \end{cases}$$

p = mean daily natural mortality rate of larval yellow perch.

$0.09 \leq p \leq 0.16$.

(S/p) = conditional event that entrained larvae would have survived in reference volume until 25th day of life, given that the mean daily natural mortality rate = p.

The probability of occurrence of the event (S/p) is:

$$\text{Prob}(S/p) = \sum_{y=1}^{25} g(y) \times e^{-p(25 - y)} =$$

$$= \frac{p_1 p_2}{p_1 p_2} \left\{ \frac{e^p(1 - p_2)}{1 - e^p(1 - p_2)} [1 - (e^p(1 - p_2))^{25}] - \right.$$

$$\left. - \frac{e^p(1 - p_2)}{1 - e^p(1 - p_1)} [1 - (e^p(1 - p_1))^{25}] \right\}$$

E(Y) = mean age of larvae upon entering the entrainment cycle =

$$= \frac{1}{p_1} + \frac{1}{p_2} - 1.$$

Prob(S/p) is defined as the fraction of larvae lost due to entrainment that would have survived to reach young-of-year stage, given that the mean age of the larval population entering the entrainment cycle is

$$E(Y) = \frac{1}{p_1} + \frac{1}{p_2} - 1,$$

and that the mean daily natural mortality rate of larvae in the reference volume is p. Table H.1 below shows the relationship between Prob(S/p) and E(Y), also plotted in Figure 45.

TABLE H.1. ESTIMATED FRACTION OF LARVAE KILLED DUE TO ENTRAINMENT THAT WOULD HAVE SURVIVED TO REACH YOUNG-OF-YEAR STAGE AS A FUNCTION OF AGE AT ENTRAINMENT

p	p_1	p_2	E(Y)	Prob(S/p)
.09	.99	.98	1.03	.116
.16	.99	.98	1.03	.020
.09	.99	.50	2.01	.127
.16	.99	.50	2.01	.026
.09	.99	.20	5.01	.177
.16	.99	.20	5.01	.055
.09	.99	.10	10.01	.241
.16	.99	.10	10.01	.116
.09	.99	.07	14.31	.26
.16	.99	.07	14.31	.125
.09	.99	.05	20.01	.24
.16	.99	.05	20.01	.12

A numerical study of larval transport within the Western Basin (10) suggests the plausibility of the geometric distribution as a model of larval residence time in basin waters prior to entrainment at the Monroe power plant, at least for larvae within a radius of a few miles of the cooling water intake. The mean residence time in basin waters prior to entrainment appears to be on the order of 1-3 days for larvae within a three mile radius of the intake and increases with linear distance from the intake. A mean age of entrained larvae of approximately 5 days is believed to be a reasonable value in the absence of data showing actual ages or lengths at the time of entrainment. The preceding calculations indicate that 17.7% of larvae that are 5 days old when entrained would have survived 25 days when the mean daily natural mortality rate is p = .09. The percentage drops to 5.5% when p = .16 (Figure 45).

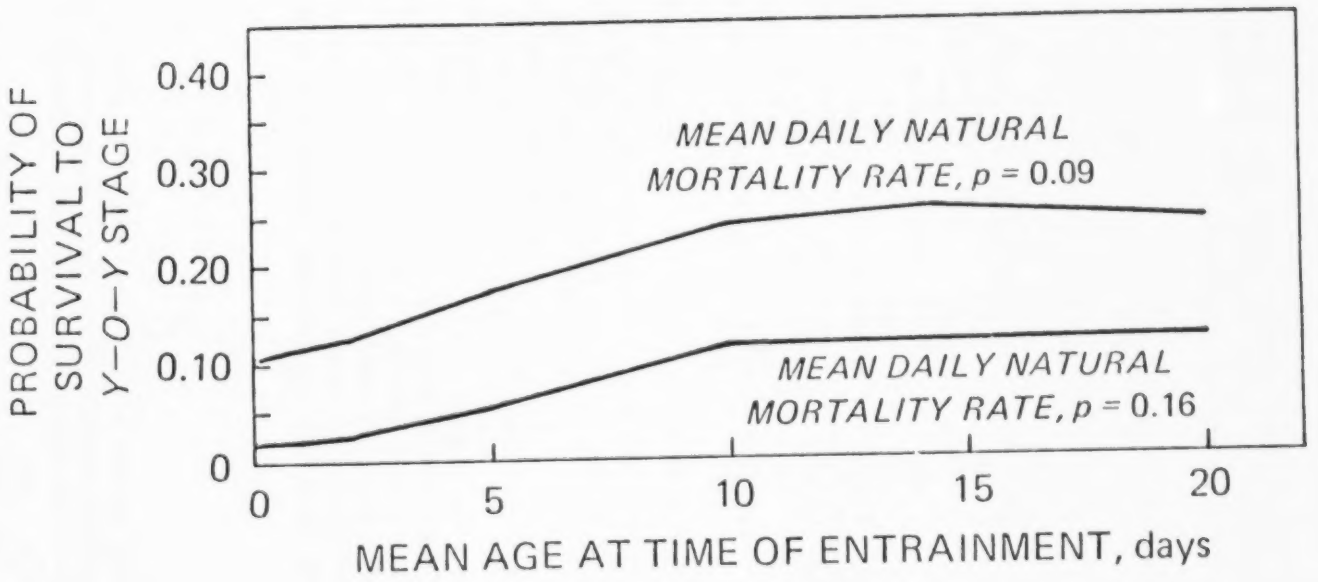

Figure 45. Plausible relationship between mean age of larvae at entrainment and fraction of larvae lost due to entrainment that would have survived to reach y-o-y stage.

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15. SUPPLEMENTARY NOTES

16. ABSTRACT

This study assessed impacts of the Monroe Power Plant upon the yellow perch population of Western Lake Erie caused by entrainment and impingement of larvae and older fishes in the plant's open cycle cooling system in 1975-76. Data analyzed in the study were collected by the Michigan Department of Natural Resources, the Center for Lake Erie Area Research of the Ohio State University, and the Institute of Water Research of Michigan State University. Estimates of total numbers of perch larvae entrained, total perch production, the natural mortality rate of perch, and the percentage of perch production that was entrained by the Monroe Power Plant were obtained for 1975-76. Impingement estimates were obtained from data supplied by the power plant. The above estimates consider only effects that occur in the same year in which entrainment and impingement occurs. Impacts may occur in subsequent years which include a depression of fish stocks and reduced yields to the fishery. Losses to the standing stocks and fisheries were estimated using a method which falls into a category known as the "equivalent-adult" type which provided estimates of the long-run annual depression of yellow perch standing stocks and the yellow perch fisheries. A numerical model was developed which incorporated several population parameters including entrainment and impingement losses, and natural mortality rates for larvae, young-of-year and juveniles, and fishing mortality rates.

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